

# OPTIMUM DESIGN OF R. C. INTZE WATER TANK

A Thesis Submitted  
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for the Degree of  
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By  
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to the  
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AUGUST, 1978

# CERTIFICATE

This is to certify that the thesis entitled  
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#### ABSTRACT

In the present work, a study of the optimum design of R.C. Intze tank is attempted. The entire superstructure and substructure of the water tower is divided into three parts viz. the tank, the supporting frame and the foundation and its optimum design is obtained interactively by solving three nonlinear programming problems sequentially.

Two problems viz. 600 KL capacity tank with 15 m staging and 1000 KL capacity tank with 22m staging are solved and the number of columns and the number of bracings at the optimum for each problem are obtained by parametric study.

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## CHAPTER 1

### GENERAL INTRODUCTION

#### 1.1 Introduction:

Outstanding progress has been achieved in the field of devising better design solutions, in the last two decades at the advent of the electronic digital computer. A conscious engineer has to investigate several alternative designs and has to choose the best among them and it is not a job of great difficulty at this age of computer. Many possibilities obviously exist, with two clearly defined extremes. One extreme is to utilize the computer capability to the fullest and 'automate' this search; the other extreme is to utilize human intuition in an 'interactive manner' to guide the computer in its calculations. Structural optimization is a broad interdisciplinary field which requires skillful combining of mathematical knowledge with engineering and it is both intellectually attractive and technologically rewarding. Hence any attempt to exploit the field of optimization for the development of better designs is worth investigating.

Extensive literature is available in the field of structural optimization as a result of simultaneous development

towards rigorous structural analysis, and efficient algorithms for optimizations. Package programs for the automated optimal design of various structures like Transmission line towers, bridges, multistorey frames have been developed and are in use. Structural optimization study for chimneys, silos, bridge decks, Transmission towers and so many others structures has been made and encouraging results have been observed. In this work an attempt is made to investigate the optimum design of reinforced concrete Intze type water tank.

## 1.2 Present Literature:

Though there are numerous papers available for the approximate and rigorous analysis for various types of tanks, the literature is practically nil towards the minimum cost design of water tank structures. But there are some general indications in selecting the type of the tanks from the various available types (6) like square or rectangular tanks, cylindrical tanks (with slab bottom or with spherical dome bottom), Intze tank etc. O.P. Jain (7) has indicated that the Intze type tank is the economical one in the capacity range between 225 KL and 1800 KL. O.P. Jain and Jai Krishna (8) have indicated that the ratio of the diameters of the cylinder and bottom ring beam should be around 0.7 and the ratio of the

cylinder diameter and the total height of the container should be around 1.0 for the economic design of the container. Approximate analysis of the staging frame for wind load (8) and for earthquake (18) are available. Charles Anthony Wilby (4) has given the state-of-the art regarding the analysis of various types of tanks (rectangular, cylindrical with or without monolithic connections) based on elastic theory and plastic theory. M.M. Basole and N.L. Khusalini (2) have developed equation to predict the cost range for cylindrical tanks.

### 1.3 Scope of the Present Work:

In this work computer programs are developed to get the minimum cost design of Intze type water tank. The study is motivated because for a given capacity and height of staging under the specified environment the number of columns and number of bracings which leads to the minimum cost design is not known and so also the geometry of the container which gives the minimum cost of the container. Furthermore, the question arises that after selecting the type of the foundation what is its configuration to get the minimum cost. The present study addresses itself to answering the questions raised above.

Chapter 2 describes the analysis of the container both for membrane and effects of continuity, the analysis

of the staging frame both for earthquake and wind forces and the analysis of the raft slab foundation.

Chapter 3 gives the detailed formulation of the optimal design problems for the container, the supporting frame and the foundation. It describes the objective function (cost of the structure) and the reasons for neglecting some variables from the design vectors. The behaviour constraints and the geometric constraints on the optimal design and a method to coalesce the bounded constraints are explained therein.

Chapter 4 gives the details of the minimization method used. A brief description of the sequential unconstrained minimization technique (SUMT), the procedure for unconstrained minimization and the one dimensional minimization method used is given in this chapter. The algorithm used to predict the initial interval of uncertainty is also explained therein. The limitations and potentialities of the developed programs are indicated. The necessary modifications required for the use of the program in some akin situations are hinted.

Chapter 5 gives the details of the numerical output of this work and the conclusions based on the study of the results of the developed programs.

## CHAPTER 2

### ANALYSIS AND DESIGN OF WATER TOWER

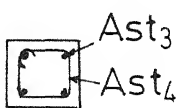
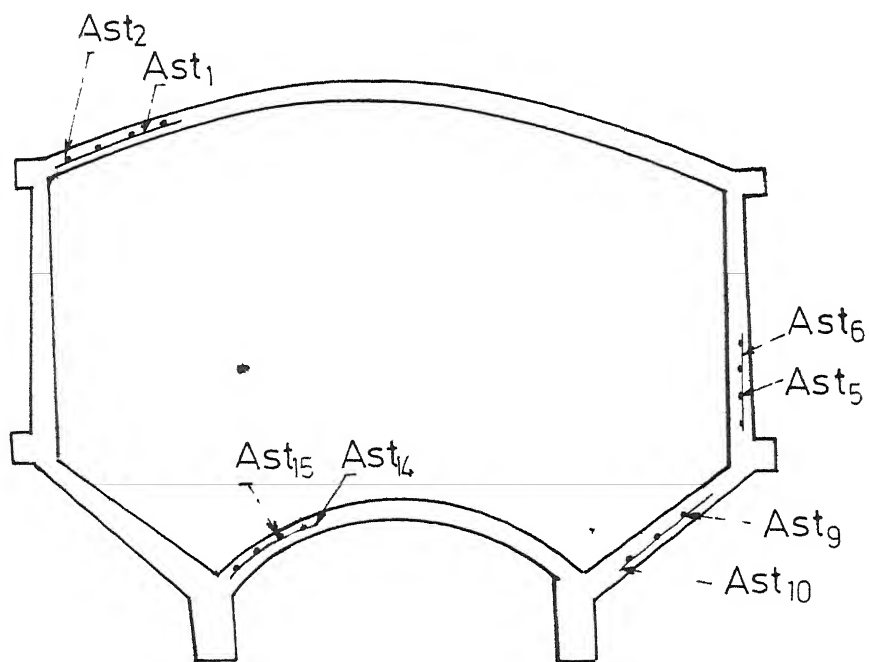
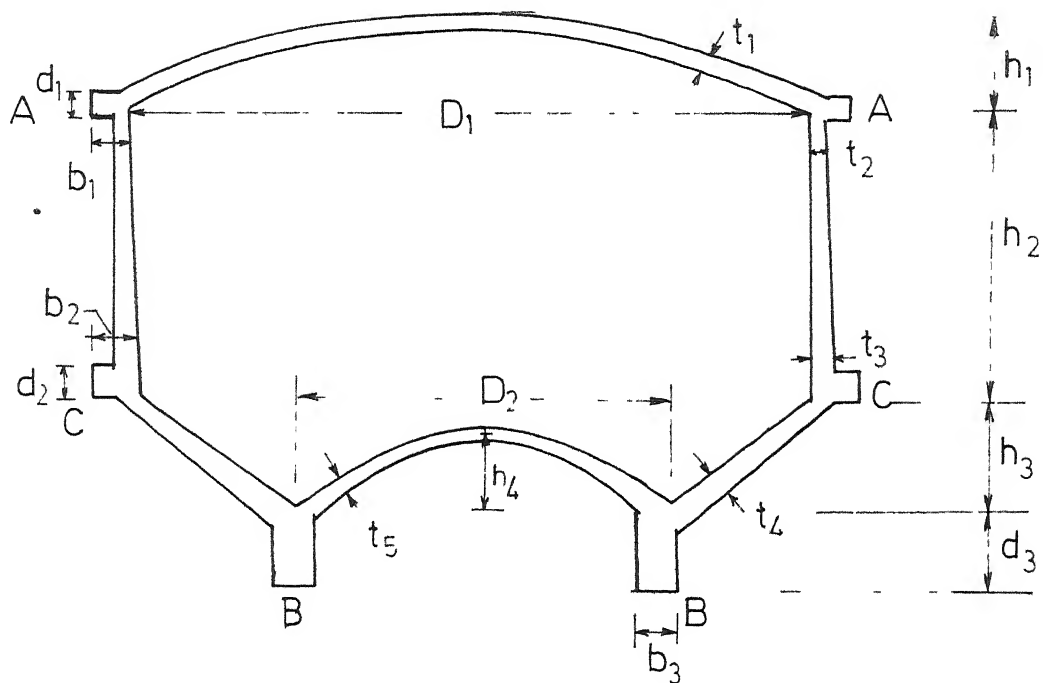
#### 2.1 Introduction:

The complete water tower structure can be considered as a combination of the three substructures viz., (i) container or tank, (ii) staging or supporting frame and (iii) foundation. The method of analysis of these substructures and the forces and moments required for the design of the various elements of the substructures are discussed in brief in this chapter.

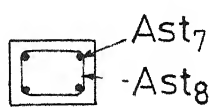
#### 2.2 Analysis and Design of the Container:

The Intze tank consists of four shells i.e. two spherical, one conical and one cylindrical shell which are connected by three ring beams as shown in Fig. 1. The loads and forces on each shell are symmetrical about the axis of revolution and the pure membrane state of stress will exist so long as each shell is simply supported at its edges i.e. it is able to undergo resulting edge displacements without restraint, while the supports supply the necessary reaction to balance the meridional forces. But in the Intze tank, at the junction of shells, the edge displacements are actually restrained. Thus the complete analysis requires both membrane

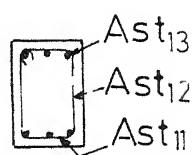




AA-RING BEAM 1



CC-RING BEAM 2



BB-RING BEAM 3

FIG 1 GEOMETRIC AND STEEL VARIABLES OF THE CONTAINER

and continuity analysis. This is well documented in (8) and a brief description follows for completeness of presentation.

### 2.2.1 Membrane analysis: (8)

(1) Top spherical shell: The top spherical dome can be designed for the meridional thrust at the edges and for the maximum hoop force.

$$\text{Meridional thrust } T_1 = wR / (1 + \cos \theta)$$

$$\text{Maximum hoop force} = wR(\cos^2 \theta + \cos \theta - 1) / (1 + \cos \theta) \text{ at } \theta = 0$$

where  $w = \text{L.L} + \text{D.L}$ .  $\theta$  is the semi central angle of the dome and  $R$  is the radius of the dome.

(2) Ring Beam 1: The meridional thrust,  $T_1$ , exerts a vertical load  $T_1 \sin \theta$  on the wall and also it imposes an outward radial force  $T_1 \cos \theta$  which causes hoop tension in the ring beam 1.

$$\text{Hoop tension} = T_1 \cos \theta D/2$$

where,  $D$  is the diameter of the tank.

(3) Cylindrical tank wall: The wall is assumed free to deform at both edges and is thus under a pure hoop tension. The maximum hoop tension can be found at the bottom and the

wall can be designed to resist this hoop tension. As the pressure varies linearly with depth, hoop tension will also vary linearly.

Hoop tension at any depth =  $p_d D/2$  , where  $p_d$  is the pressure at depth  $d$ .

(4) Ring beam 2: If the weight transmitted through the tank wall at top of conical dome is  $w$  per unit length and  $\theta$  is the inclination of the dome with the vertical, then the meridional thrust  $T_1$  caused is  $w \cdot \sec \theta$  per unit length of circumference.

Outward radial force in ring beam 2 =  $T_1 \sin \theta = w \tan \theta$

Therefore, Hoop tension in ring beam 2 =  $w \cdot \tan \theta \cdot D/2$

The ring beam should be designed for the hoop tension calculated as above.

(5) Conical dome : The conical dome supports a uniform vertical load from walls at its top edge. At top of the dome a hoop tension will be created which exerts a radial inward force at each slanting strip (dome is assumed as consisting of individual slanting strips monolithically joined) and will oppose its rotation outward. The magnitude of the radial force created at top edge is so much that on combining with the vertical load, the resultant lies along the meridian of the conical dome. Thus the vertical load at top edge of

the conical dome is supported by it with the creation of meridional thrust and a hoop tension. The water pressure on conical dome and its own weight acting at any point give rise to hoop tensions at each plane, whose inward reaction, together with the water pressure and weight of dome, cause a resultant force which is meridional.

Meridional thrust at any height  $d = W_v / (\pi D' \cos \theta)$  where  $W_v$  is the total vertical load acting on the dome above that height and  $D'$  is the diameter at that level. These expressions are used to calculate the necessary forces to design the sections of the conical dome.

(6) Bottom dome: Like the top spherical dome, the bottom dome also develops only compressive stresses both meridionally and along hoops. The maximum meridional thrust at the edges and maximum hoop force (at  $\theta = 0$ ) can be calculated as described earlier.

(7) Ring Beam 3 : This beam receives an inward inclined thrust from the conical dome and inclined outward thrust due to the reaction from the bottom spherical dome. The vertical components of these two thrusts add up and the ring beam is designed for this load. Their horizontal components oppose each other and depending upon their relative magnitudes, ring beam is either in hoop tension or in hoop compression and can

be designed accordingly.

### 2.2.2 Continuity analysis:<sup>(8)</sup>

The forces due to continuity are obtained by applying the principle of consistent deformations. The vertical displacements are always consistent at each joint as each shell is free to deform in this direction and consistency has only to be satisfied for horizontal and angular displacements between the shells meeting at a joint. Based on the stiffness of each shell at edges for horizontal and angular moments, equations of consistency are framed and the forces due to continuity can be determined after solving this system of simultaneous equations in rotation ( $\delta$ ) and displacement ( $x$ ).

Membrane deformations and stiffnesses at the edges of various elements can be expressed as follows:

(1) Top dome: Slope at left edge  $\delta_d = 2wR \sin \theta / Et$  (clockwise)

Horizontal deflection  $x_d = \frac{wR^2 \sin^2 \theta}{Et} (1/(1+\cos \theta) - \cos \theta)$  (Inwards)

Moment stiffness per unit length  $M_1 = \frac{REt}{4\lambda^3} (k + \frac{1}{k})$

Corresponding radial force per unit length  $H_1 = Et/(2\lambda^2 k \sin \theta)$

Thrust stiffness per unit length  $H_2 = Et/(\lambda R k \sin^2 \theta)$

Corresponding moment per unit length  $M_2 = Et/(2\lambda^2 k \sin \theta)$

where ,  $\lambda^4 = 3 \left( \frac{R}{t} \right)^2$  ;  $k = 1 - \frac{1}{2\lambda} \cot \theta$

(2) Ring beam 1: Radial thrust to cause unit outward deflection  $H_3 = \frac{E b d}{R^2}$

Moment per unit circumference to cause unit rotation

$$M_3 = \frac{E b d^3}{12 R^2}$$

(3) Cylindrical wall: Moment stiffness  $M_4 = 2\mu z$

Corresponding thrust  $H_4 = 2\mu^2 z$

Thrust stiffness  $H_5 = 4\mu^3 z$

Corresponding moment  $M_5 = 2\mu^2 z$

Displacement of tank wall at bottom  $x_w = \frac{p R^2}{E t}$

Slope of the wall  $\delta_w = \frac{p R^2}{E t h}$

(4) Ring beam 2: Radial thrust to cause unit outward

deflection  $H_6 = \frac{E b d}{R^2}$

Moment per unit circumference to cause unit rotation  $M_6 = \frac{E b d^3}{12 R^2}$

where, b and d are breadth and depth of the beam and R is the radius of the cylindrical shell.

(5) Conical dome : Outward deflection at the top edge =  $\frac{D \cdot H_t}{2 E t}$

Outward deflection at the bottom edge =  $\frac{D \cdot H_b}{2 E t}$

$$\text{Slope at the edges} = \frac{\tan \theta}{Et} (2T_2 - T_1)$$

Stiffnesses at the top edge:

$$\text{Moment stiffness } M_7 = \frac{Et \cdot k_4 \cdot l}{(k_1 \cdot k_4 - k_2 k_3) \tan^2 \theta}$$

$$\text{Corresponding thrust } H_7 = \frac{Et \cdot k_2}{(k_1 k_4 - k_2 k_3) \sin \theta \tan \theta}$$

$$\text{Thrust stiffness } H_8 = \frac{Et \cdot k_1}{l \sin^2 \theta (k_1 k_4 - k_2 k_3)}$$

$$\text{Corresponding moment } M_8 = \frac{Et \cdot k_3}{\sin \theta \tan \theta (k_1 k_4 - k_2 k_3)}$$

Stiffnesses at the bottom edge:

$$\text{Moment stiffness } M_9 = \frac{Et \cdot k_4 \cdot l}{(k'_1 k'_4 - k'_2 k'_3) \tan^2 \theta}$$

$$\text{Corresponding thrust, } H_9 = \frac{Et \cdot k'_3}{(k'_1 k'_4 - k'_2 k'_3) \sin \theta \tan \theta}$$

$$\text{Thrust stiffness } H_{10} = \frac{Et \cdot k'_1}{l \sin^2 \theta (k'_1 k'_4 - k'_2 k'_3)}$$

$$\text{Corresponding moment } M_{10} = \frac{Et \cdot k'_3}{(k'_1 k'_4 - k'_2 k'_3) \sin \theta \tan \theta}$$

For the given geometry of the conical shell  $k_1, k_2, k_3, k_4$  and

$k'_1, k'_2, k'_3, k'_4$  can be obtained from the Table 1 shown, after evaluating  $\epsilon, \epsilon'$  as below.  $l$  is the slant length of the cone (complete) and  $l'$  is the truncated cone slant length.  $\Delta^4 = \frac{12 \cot^2 \theta}{t^2}$  where  $\theta$  is the semi apex angle of the cone.

$$\epsilon = 2\Delta\sqrt{l} \quad \text{and} \quad \epsilon' = 2\Delta\sqrt{l'}$$

$$(6) \quad \text{Ring beam 3: Angular rotation } \phi = \frac{12 R^2}{bd^3 E} \left( m - H \frac{d}{2} \right)$$

where,  $m$  is the clockwise edge moment per unit circumference length applied to the beam at its top and  $H$  is the outward radial force.  $b$  and  $d$  are the breadth and depth of the ring beam. Inward moment of top edge due to an angular rotation  $= \phi \frac{d}{2}$

$$\text{Outward moment due to hoop tension} = \frac{H R}{bd E} R$$

$$\text{Equating these two, } \phi = \frac{3R^2}{bd^3 E} m$$

$$\text{Moment stiffness, } M_{11} = \frac{bd^3 E}{3R^2} \text{ per radian}$$

$$\text{Corresponding outward thrust } H_{11} = \frac{bd^2 E}{2R^2} \text{ per radian}$$

$$\text{Thrust stiffness } H_{12} = \frac{bdE}{R^2} \text{ per unit movement}$$

$$\text{Corresponding edge moment } M_{12} = \frac{bd^2 E}{2R^2} \text{ per unit movement}$$

$$(7) \quad \text{Bottom dome: Slope at edges } \delta = \frac{2 w R \sin \theta}{Et} - \frac{w_p R^2 \sin \theta}{Et}$$



TABLE 1 : CONICAL SHELL STIFFNESS CONSTANTS

$\epsilon$ or $\epsilon'$	$K_1$	$K_2, K_3$	$K_4$	$K'_1$	$K'_2, K'_3$	$K'_4$
10	183.0	25.10	6.79	169.0	23.55	7.30
11	220.5	27.25	7.21	236.5	29.35	7.90
12	288.0	32.60	7.91	312.0	36.00	8.70
13	366.5	38.40	8.60	396.0	42.20	9.40
14	463.0	46.20	9.36	490.0	48.85	10.09
15	568.5	52.50	10.10	596.0	56.30	10.82
16	696.0	60.30	10.82	728.0	63.20	11.42
17	834.0	68.40	11.52	880.0	72.30	12.13
18	990.0	76.65	12.24	1038.0	81.20	12.97
19	1172.0	86.00	12.98	1222.0	89.70	13.61
20	1370.0	95.90	13.69	1430.0	100.00	14.30
21	1574.0	105.60	14.39	1658.0	111.0	15.10
22	1830.0	116.3	15.10	1900.0	121.1	15.70
23	2100.0	127.8	15.88	2190.0	134.0	16.50
24	2400.0	139.9	16.45	2470.0	145.0	17.16
25	2695	175.0	17.24	2800.0	156.9	17.80

$$\text{Horizontal moment of edges } x = \frac{wR^2 \sin \theta}{Et} \left( \frac{1}{1+\cos \theta} - \cos \theta \right) \\ - \frac{w_r R^2 \sin \theta}{2 Et} + \frac{w_p R^3 \sin \theta}{6 Et(1+\cos \theta)} (2\cos 2\theta + \cos \theta - 3)$$

$$\text{Moment stiffness } M_{13} = \frac{REt}{4\lambda^3} \left( k + \frac{1}{k} \right)$$

$$\text{Corresponding thrust } H_{13} = Et/(2\lambda^2 k \sin \theta)$$

$$\text{Thrust stiffness } H_{14} = Et/(2\lambda^2 k \sin \theta)$$

$$\text{Corresponding moment } M_{14} = Et/(2\lambda^2 k \sin \theta) \text{ where, } \lambda^4 = 3(R/t)^2 \text{ and} \\ k = 1 - \frac{1}{2\lambda} \cot \theta .$$

Reactions due to continuity at ring beam 1 junction:

Let the net rotation of the joint be  $\delta_1$  clockwise and net displacement be  $x_1$  inward. Then the changes in the slope and displacement from the membrane states can be expressed as follows:

Element	Slope	Displacement
Top dome	$\delta_1 - \delta_d$	$x - x_d$
Ring beam	$\delta_1$	$x_1$
Tank wall	$\delta_1 - \delta_w$	$x_1$

where,  $x_d$ ,  $x_w$  displacements of dome and wall respectively and  $\delta_d$ ,  $\delta_w$  slopes of dome and wall respectively.

Equations of consistency of deformations (Joint at Ring beam 1):

$$M_1(\delta_1 - \delta_d) + H_1(x_1 - x_d) + M_3\delta_1 + M_4(\delta_1 - \delta_w) + H_4x_1 = 0$$

$$H_2(x_1 - x_d) + M_2(\delta_1 - \delta_d) + H_3x_1 + H_4(\delta_1 - \delta_w) + H_5x_1 + (\text{thrust from dome}) = 0$$

These two equations can be solved for  $\delta_1$  and  $x_1$  and the reactions due to continuity can be evaluated.  $R_1$  to  $R_9$  are the reactions derived from these evolution.

Element	Moment	Thrust	Hoop tension
Dome	$R_1$	$R_4$	$R_7$
Ring beam 1	$R_2$	$R_5$	$R_8$
Wall	$R_3$	$R_6$	$R_9$

$$R_1 = M_1(\delta_1 - \delta_d) + (x_1 - x_d) H_1; R_2 = M_3\delta_1; R_3 = M_4(\delta_1 - \delta_w) + H_4x_1$$

$$R_4 = H_2(x_1 - x_d) + M_2(\delta_1 - \delta_d); R_5 = H_3x_1; R_6 = H_4(\delta_1 - \delta_w) + H_5x_1$$

$$R_7 = \frac{x_1 E A_1}{(D/2)}; R_8 = -\frac{x_1 E A_2}{(D/2)}; R_9 = \frac{x_1 E A_3}{(D/2)};$$

where  $A_1, A_2, A_3$  are the cross sectional area of the respective member for unit length .

Reactions due to continuity at the ring beam 2 joint:

Let  $\delta_2$  and  $x_2$  be the rotation and displacement of this joint, then changes in the slope and deformation of the

elements and their reactions can be expressed as follows:

Element	Slope	Displacement	Moment	Thrust	Hoop Tension
Wall	$\delta_2 - \delta_w$	$x_2 - x_w$	$R_{10}$	$R_{13}$	$R_{16}$
Ring beam	$\delta_2$	$x_2$	$R_{11}$	$R_{14}$	$R_{17}$
Conical dome	$\delta_2 - \delta_c$	$\delta_2 - \delta_c$	$R_{12}$	$R_{15}$	$R_{18}$

$\delta_w, x_w$  - slope and displacement of the wall

$\delta_c, x_c$  - slope and displacement of the conical dome

$$M_4(\delta_2 - \delta_w) + H_4(x_2 - x_w) + M_6\delta_2 + M_7(x_2 - x_c) + \text{Moment due to balcony slab} = 0$$

$$M_5(\delta_2 - \delta_w) + H_5(x_2 - x_w) + H_6x_2 + M_8(\delta_2 - \delta_c) + H_8(x_2 - x_c) = 0$$

These two equations can be solved for  $\delta_2$  and  $x_2$  and the reactions can be evaluated as shown below.

$$R_{10} = M_4(\delta_2 - \delta_w) + H_4(x_2 - x_w) ; R_{11} = M_6\delta_2 ; R_{12} = M_7(x_2 - x_c)$$

$$R_{13} = M_5(\delta_2 - \delta_w) + H_5(x_2 - x_w) ; R_{14} = H_6x_2 ; R_{15} = M_8(\delta_2 - \delta_c) + H_8(x_2 - x_c)$$

$$R_{16} = \frac{x_2 EA_4}{(D/2)} ; R_{17} = \frac{x_2 EA_5}{(D/2)} ; R_{18} = \frac{x_2 EA_6}{(D/2)}$$

where  $A_4, A_5, A_6$  are the cross sectional areas of the respective elements for unit length.

Reactions due to continuity at the ring beam 3 joint:

Let the rotation and displacement of this joint be  $\delta_3$  and  $x_3$  then the changes in slope and displacement of the elements (from the membrane state) and their reactions can be expressed as follows.

Element	Slope	Displacement	Moment	Thrust	Hoop tension
Conical Dome	$\delta_3 - \delta_c$	$x_3 - x_c$	$R_{19}$	$R_{22}$	$R_{25}$
Ring beam 3	$\delta_3$	$x_3$	$R_{20}$	$R_{23}$	$R_{26}$
Bottom dome	$\delta_3 - \delta_b$	$x_3 - x_b$	$R_{21}$	$R_{24}$	$R_{27}$

$\delta_b$  and  $x_b$  represents slope and displacement of the bottom dome respectively.

$$M_9(\delta_3 - \delta_c) + H_9(x_3 - x_c) + M_{11}\delta_3 + H_{11}x_3 + M_{13}(\delta_3 - \delta_b) + H_{13}(x_3 - x_b) = 0$$

$$H_{10}(\delta_3 - \delta_c) + H_{11}(x_3 - x_c) + M_{12}\delta_3 + H_{12}x_3 + M_{14}(\delta_3 - \delta_4) + H_{14}(x_3 - x_b)$$

$$+ (\text{Thrust from both the domes}) = 0$$

These equations can be solved for  $x_3$  and  $\delta_3$  and the reactions  $R_{19}$  to  $R_{27}$  can be calculated.

$$R_{19} = M_9(\delta_3 - \delta_c) + H_9(x_3 - x_c) ; R_{20} = M_{11}\delta_3 + H_{11}x_3$$

$$R_{21} = M_{13}(\delta_3 - \delta_b) + H_{13}(x_3 - x_b) ; R_{22} = H_{10}(\delta_3 - \delta_c) + H_{11}(x_3 - x_c)$$

$$R_{23} = M_{12}\delta_3 + H_{12}x_3 ; R_{24} = M_{14}(\delta_3 - \delta_b) + H_{14}(x_3 - x_b)$$

$R_{25}$ ,  $R_{26}$  and  $R_{27}$  can be calculated using the corresponding unit length areas in  $\frac{x_3 EA}{(D_2/2)}$  as before, where  $D_2$  is the diameter of the ring beam 3.

The continuity alters the hoop forces near joints and imposes meridional moments and they die out after some distance and hence they are of local nature maintaining the meridional stresses in the interior portions unaltered. The design will be improved by providing sufficient reinforcements and thickening the shell edges, based on the above calculated reactions.

#### Analysis of the ring beam 3:

The load of the tank is transferred to the columns through this circular beam. This beam is subjected to a uniformly distributed load all along its length. The following expressions are used to calculate the design moments of the circular beam.

$$\text{Support moment} = \beta wR^2(2\alpha)$$

$$\text{Mid-span moment} = \beta' wR^2(2\alpha)$$

$$\text{Maximum twisting moment} = \beta'' wR^2(2\alpha)$$

where  $2\alpha$  is the angle subtended at the centre by two column points (ie.)  $2\alpha = (\pi/n_c)$  radians, where  $n_c$  - number of columns.

$$\beta = (1 - \alpha \cot \alpha) / 2\alpha ; \quad \beta' = (-1.0 + \alpha \sin \alpha + \cos^2 \alpha / \sin \alpha) / 2\alpha$$

$$\beta'' = (\phi - \alpha + \alpha \cos \phi - \alpha \cot \alpha \alpha_s) / 2\alpha , \text{ where } \alpha_s = \sin^2 \alpha - \cos \alpha (\alpha^2 - \sin^2 \alpha)^{1/2}$$

$$\text{and } \phi = \sin^{-1}(\alpha_s)$$

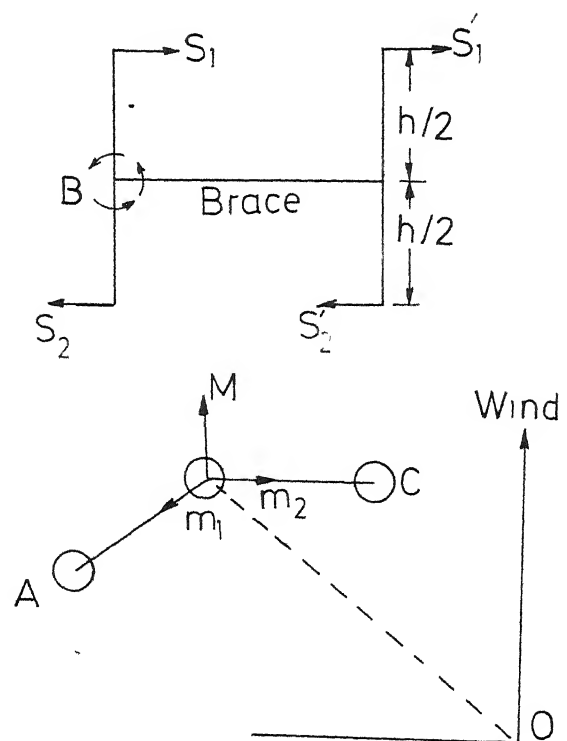
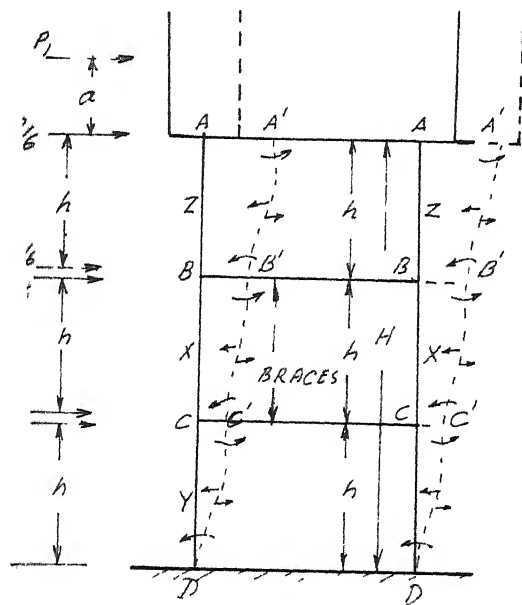
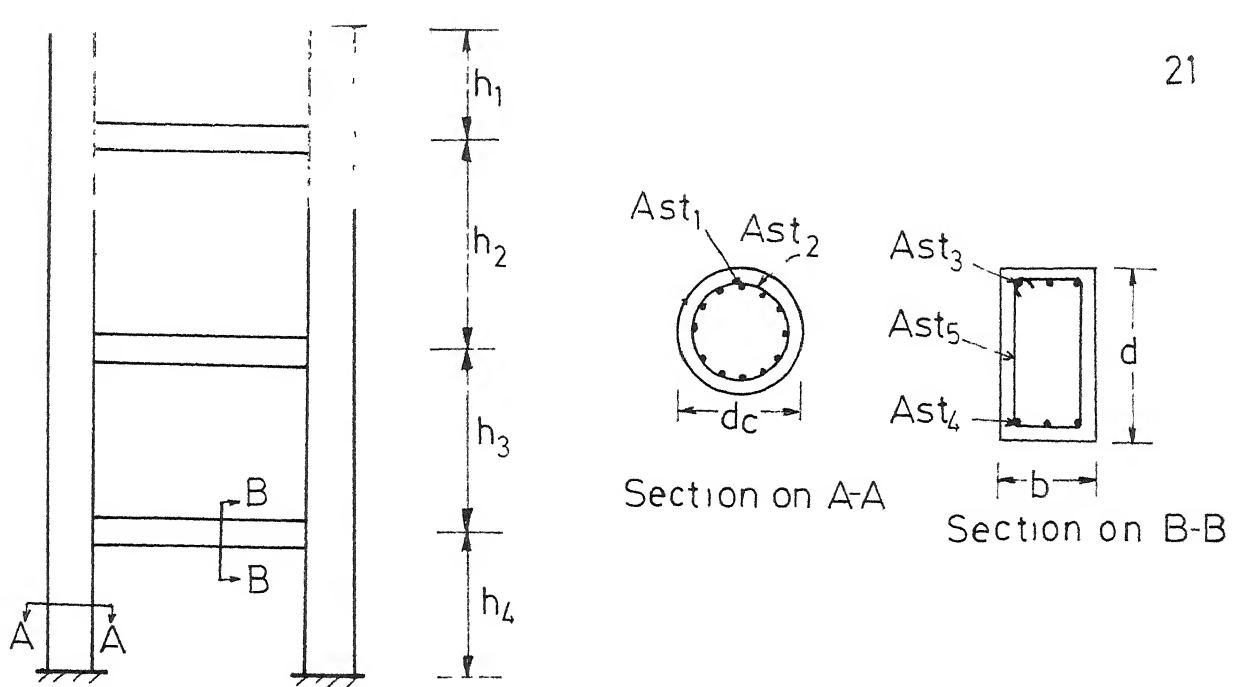
$\beta, \beta', \beta''$  are the curved beam coefficients which depends on the number of columns (i.e.)  $\alpha$  at the centre.

### 2.3 Supporting Frame Analysis:

The supporting frame consists of R.C. columns, placed symmetrically on the circumference of a circle and braced together by horizontal members as shown in Fig.2. The columns are subjected to the vertical load and the lateral forces due to wind or earthquakes, acting on the tank and the frame. The problem of analysis can be done rigorously by considering it as a space frame but in the normal design procedure and in the situations where repeated analysis is required one should go in for the approximate method of analysis.

#### 2.3.1 Wind load analysis: <sup>(8)</sup>

Since the tank base, foundation and the braces are very stiff compared to the columns it is considered that the tower deflects, maintaining the column axes almost vertical at their top, bottom and at their junctions with braces developing the points of inflexion at mid heights of each





panel. The internal reactions at these inflexion points can be calculated as described below. The deflected pattern of the tower is shown in Fig.2(a).

Considering the total cross sectional area of all the columns as the area of circular ring, moment of inertia of the assumed cantilever beam of circular ring section  $= \frac{\pi n_c^3 t}{8} = \frac{n_c a D^2}{8}$

Therefore, vertical force on the farthest leeward side

$$\text{column} = \frac{4m}{n_c a D} = \frac{4m}{n_c D} \quad \left( \text{Because } \frac{m}{\frac{n_c a D^2}{8}} \frac{D}{2} = \text{stress} \right)$$

where, m - moment in the assumed cantilever beam,  $n_c$  - number of columns, a - area of one column, D - diameter. Vertical force in columns lying on bending axis is zero.

Shear stress in the column (at  $\theta$  from the bending axis)

$$q = \frac{Q}{2t \sec \theta n_c a D^2/8} 2(D/2)^2 t \cos \theta = \frac{2Q \cos^2 \theta}{n_c a}$$

Shear force maximum (at  $\theta = 0$ )  $S = q_0 a = \frac{2Q}{n_c}$ , where

Q is the lateral force acting at that level.

The maximum moment will occur at the top and bottom end of each panel and will be equal to  $S \frac{h}{2}$  where h is the clear panel height. Thus these forces and moments (i.e.)

vertical load, bending moment and shear force can be used for the design of columns.

#### Analysis of Bracing beams:

The shear forces acting on either side and the moments set up at the joint are shown in the Fig.2(h). Considering the static equilibrium of moments at the joint,

$$\frac{m_1}{\sin(\theta+\pi/n_c)} = -\frac{m_2}{\sin(\theta-\pi/n_c)} = \frac{M}{\sin(2\pi/n_c)} \quad \text{is the equation}$$

relating  $m_1$ ,  $m_2$  and  $M$ . But  $= \frac{(Q_1 h_1 + Q_2 h_2)}{n_c} \cos^2 \theta$  where  $Q_1, Q_2$

and  $h_1, h_2$  denote shear forces and panel heights above and below the bracing, respectively.

Design forces and moments for the bracing beam:

$$\text{Maximum moment} = \frac{M_{\max}}{\sin(2\pi/n_c)} \sin(\theta_{\max} + \pi/n_c) \quad \text{where,}$$

$$M_{\max} = M \text{ at } \theta = \theta_{\max}.$$

$\theta = \theta_{\max}$  can be obtained from the condition for maximum moment  $m_1$  (i.e.)  $\theta$  which satisfies  $2 \tan \theta \tan(\theta + \pi/n_c) = 1$ .

$$\text{Shear force maximum} = \frac{Q_1 h_1 + Q_2 h_2}{AB n_c \sin(2\pi/n_c)} (\cos^2 \theta \sin(\theta + \pi/n_c) - \cos^2(\theta - 2\pi/n_c) \sin(\theta - 3\pi/n_c))$$

$$\sin(\theta - 3\pi/n_c))$$

where,  $\theta = \pi/n_c$  and  $AB$  is the length of the bracing beam.

The twisting moment can be assumed as 5 percent of the maximum bending moment.

### 2.3.2 Earthquake load analysis: <sup>(18)</sup>

The supporting tower should be designed for the forces and moments due to earthquake loading also. The design forces can be obtained by any one of the two methods specified in IS:1893 (criteria for earthquake resistant design of structures) i.e. 1. Seismic coefficient method. Response spectrum method. In the later one the earthquake factor is calculated and the earthquake force is obtained which is coming as shear in the columns.

$$\text{Earthquake factor } \alpha_h = \beta I F_o \frac{S_a}{g}$$

$\beta$  = coefficient depending upon the soil-foundation system

$I$  = coefficient depending upon the importance of the structure

$F_o$  = seismic zone factor for average acceleration spectra as given in Table 2 of IS:1893.

$\frac{S_a}{g}$  = average acceleration coefficient depending on the damping and appropriate natural period.

$I = 1.5$ ,  $\beta = 1.0$ ,  $F_o = 0.2$  are the coefficients considered in this work and 5 percent damping is considered as per 5.2.2 IS:1893 specification for elevated tower supported tanks.

Natural period of vibration  $T = 2\pi\sqrt{A/g}$  sec.

where ,  $\Delta = \int \frac{mM}{EI} dx$  ;     E - Youngs modulus of concrete

I - Moment of inertia of the column

Earthquake force =  $\alpha_h$  \* (Total load + 1/3 Self load)

Shear force per column = (Earthquake force)/Number of columns

Moment in the column = shear force \*  $\frac{h_c}{2}$  where  $h_c$  - clear panel height  
( $M_{eq}$ )

Reaction force in the column due to the moment at that level can be obtained by  $F = \frac{M}{\Sigma r^2} R$  and then the direct load on

the column is added to it to get the total load  $P_{eq}$  coming on the column. The column should be designed for these two forces  $M_{eq}$  and  $P_{eq}$ . The moment in the bracing beam can be calculated from  $M_{eq}$  and the beam can be designed accordingly.

Moment in the bracing beam  $M_{br} = \frac{M_{eq}}{\cos(\pi/2 - \pi/n_c)}$  , where  $n_c$  denotes

the number of columns.

## 2.4 Foundation Analysis and Design:

Depending on the soil characteristics the selection of the type foundation can be made. There are various types of foundation which can be provided for water tank structure. A combined raft foundation with complete raft slab and the foundation with ring raft slab(i.e.) slab combined with the bottom circular beam, are the two types among them. Pile foundation is also sometimes used. The analysis of ring type of raft



as shown in Fig. 3 which is selected in the present work is briefly explained below:

The uniformly distributed load coming on the circular beam is calculated. Then the design moments (i.e.) support moment, midspan moment and twisting moment can be calculated using the circular beam coefficients  $\beta$ ,  $\beta'$  and  $\beta''$  as shown earlier. The raft slab can be considered as a cantilever slab with the soil reaction as the uniformly distributed loading. The maximum negative moment can be calculated and the slab can be designed for these forces. The reinforcements can be curtailed at the half distance keeping the alternative bars to the complete length, and the distribution steel can be provided. The actual pressure of the soil should be less than the allowable bearing capacity of the soil. This analysis is an approximate one and is quite suitable for embedment in the optimal design routine because it is simple and reasonably accurate and will not consume more time during the function and gradient evaluation.

## 2.5 Summary:

The membrane analysis and continuity analysis of the tank, approximate method of analysis of the staging frame for wind forces and earthquake forces, and the analysis of raft slab foundation have been explained in this chapter. These are embedded in the optimization programs explained in the succeeding chapter.

## CHAPTER 3

### OPTIMAL DESIGN FORMULATION

#### 3.1 Introduction:

The structural optimization problem can be formulated either using mathematical programming approach or the optimality criteria approach. The present problem of optimizing the design for minimum cost of a water tower structure is formulated as a mathematical programming problem. The design vectors, constraints and the formulation of the objective functions of each problem are explained in this chapter.

The one-shot formulation of the optimum design of the entire water tank structure becomes a complex problem due to the following facts.

1. Since the optimum solution seeking is a repetitive analysis-design procedure the analysis of the composite structure viz. the analysis of the container with membrane and continuity effects, the frame analysis of the supporting tower for wind and earthquake loadings and the foundation analysis is time-consuming procedure.
2. Since the entire water tank structure consists of numerous variables, the problem becomes a large

non-linear programming problem which requires an efficient solution technique, sufficient computer memory and more computer time for seeking the solution.

3. Since the number of columns and the number of bracings influence the entire analysis-design procedure, as well as the size of the design vector and the problem formulation, it is difficult to include them in an automated optimum design procedure.

4. Since the type of the foundation differs for different soil conditions it is difficult to formulate the problem including the foundation unless the analysis-design of the various foundations are developed with the proper strategy to select the foundation.

Therefore, in the present work optimum design of the water tank is obtained by substructuring the problem into three parts viz. the tank, the staging and the foundation and solving them sequentially.

### 3.2 Optimum Design Formulation of the Container:

The geometry of the container of the intze tank is shown in Fig.1,  $t_1, t_2, t_3, t_4, t_5$  are the thickness of the top dome, wall thickness at top edge, wall thickness at bottom edge, conical dome thickness and the bottom dome thickness



respectively.  $D_1$  and  $D_2$  are the diameters as shown in Fig.1.  $b_1, d_1, b_2, d_2, b_3, d_3$  are breadths and depths of the Ring beam 1, 2, 3 respectively and  $h_1, h_2, h_3, h_4$  are the heights and rise of the domes as shown in the Fig.1.

The various steel reinforcements are also shown . Thus there are fifteen steel variables and seventeen geometric variables in the problem. To reduce the complexity of the problem the design vector is considered to have only fifteen variables. The reasons for neglecting the remaining variables from the design vector are explained below:

(1) The top dome carries its own weight and the nominal live load and in general the design comes out with the minimum requirements of steel and thickness as per IS code.

(2) The geometry (breadth and depth) of the top ring beam can be assessed judiciously which depends on the hoop tension on the ring beam from the top dome. The reinforcements can be provided for this hoop tension based on the allowable stress in steel.

(3) In order to reduce one variable due to the variation in the cylindrical wall, the bottom edge thickness is considered as a design variable prescribing minimum thickness based on code requirement at the top edge.

(4) Curved beam coefficients shows that the maximum positive bending moment at the mid span is approximately

equal to half of the support moment. Since the cross sections of the circular beam at the support section and at the midspan section are same, half of the reinforcements provided for the midspan moment will be bent and placed at top of the beam from which it can be seen that the beam will have equal area of steel both at top and bottom at the support section. This relation between them facilitates to remove one steel variable from the design vector.

(5) The rise of the top dome and the bottom dome can be fixed as  $1/8$ th of the cylinder diameter and  $1/6$ th of the bottom dome diameter to avoid  $h_1$  and  $h_4$  from the design vector.

(6) The reinforcements in the bottom dome generally comes as per minimum requirement and hence they are neglected from the design vector.

(7) The steel variables like the distribution steel, stirrups can be assessed properly based on the code requirement, and hence they are not considered as design variables.

Thus the above variables which seems to have negligible effect on the cost of the structure are neglected. The design variables considered in the problem are listed below (Ref.Fig.1).

### 3.2.1 Design variables:

- $X_1$  - Diameter of the tank ( $D_1$ )
- $X_2$  - height of the cylindrical shell ( $h_2$ )
- $X_3$  - thickness of the cylindrical shell at the bottom edge( $t_3$ )
- $X_4$  - breadth of the ring beam 2 ( $b_2$ )
- $X_5$  - depth of the ring beam 2 ( $d_2$ )
- $X_6$  - thickness of the conical shell ( $t_4$ )
- $X_7$  - height of the conical shell ( $h_3$ )
- $X_8$  - diameter of the conical dome ( $D_2$ )
- $X_9$  - breadth of the ring beam 3 ( $b_3$ )
- $X_{10}$  - depth of the Ring beam 3 ( $d_3$ )
- $X_{11}$  - thickness of the bottom dome ( $t_5$ )
- $X_{12}$  - area of hoop steel at the cylindrical shell bottom edge  
( $Ast_5$ )
- $X_{13}$  - area of hoop steel at the Ring beam 2 ( $Ast_7$ )
- $X_{14}$  - area of hoop steel at the conical dome ( $Ast_9$ )
- $X_{15}$  - area of steel at the support section of the ring  
beam 3 ( $Ast_{11}$ ).

The area of steel in each case represents the fraction of respective cross-section of the elements.

### 3.2.2 Objective function:

The objective function is the cost of the tank which is the function of the design variables. The total cost of the tank

can be expressed as the sum of the cost of concrete, steel and shuttering.

$$\begin{aligned}\text{Total cost } F(X) &= \text{Cost of concrete } C_1 + \text{Cost of steel } C_2 \\ &\quad + \text{Cost of shuttering } C_3 \\ &= V_c C_c + W_s C_s + A_c C_{sh}\end{aligned}$$

where,  $C_c$ ,  $C_s$ ,  $C_{sh}$  denotes cost of concrete per  $m^3$ , cost of steel per tonne and cost of shuttering per  $m^2$  respectively.  $V_c$ ,  $W_s$ ,  $A_c$  denotes volume of concrete, weight of steel and total shuttering area. Since the cost of shuttering varies for conical dome and spherical dome (from the cost for cylindrical wall shuttering) proper weight factors have been considered while calculating the over all shuttering area  $A_c$  in terms of straight shuttering area.

(i.e.) Cost of shuttering for conical dome  $= \alpha_c C_{sh}$  per  $m^2$

Cost of shuttering for spherical dome  $= \alpha_d C_{sh}$  per  $m^2$

$\alpha_c = 1.25$  and  $\alpha_d = 1.5$  are the weight factors considered

in the present work

$$V_c = \pi D_1 h_2 t_3 + \pi (D_1 + b_1) b_2 d_2 + \pi (D_1 + D_2) \left( h_3^2 + \left( \frac{D_1 - D_2}{2} \right)^2 \right) t_4$$

$$+ \pi 2.0.833.D_2 \cdot \frac{1}{6} h_2 \cdot t_5 + \pi D_2 \cdot b_3 \cdot d_3 + \pi D_1^2 t_1 / 4 + \pi D_1 b_1 d_1$$

= the sum of volumes of various elements (i.e.) top dome, Ring beam 1, cylindrical wall, Ring beam 2, conical dome, Ring beam 3 and conical dome.

$$\begin{aligned}
I_s = & \frac{\pi D_1^2}{4} (Ast_1 + Ast_2) + \pi D_1 b_1 d_1 (Ast_3 + Ast_4) + \pi D_1 \frac{t_1 + t_2}{2} h_1 (Ast_5 + Ast_6) \\
& + \pi D_1 b_2 d_2 (Ast_7 + Ast_8) + \pi \left( \frac{D_1 + D_2}{2} \right) \left( h_3^2 + \left( \frac{D_1 - D_2}{2} \right)^2 \right)^{1/2} t_4 (Ast_9 + Ast_{10}) \\
& + \pi D_2^2 \frac{1}{6} 0.833 t_5 + b_3 d_3 \pi D_2 \left( 2 Ast_{11} + \frac{1.5}{2} Ast_{12} \right) \\
A_c = & 2\pi (D_1 + t_3) h_2 + \alpha_d \left( \pi D_1^2 / 4 + 2\pi 0.833 D_2^2 / 6.0 \right) + \left( \frac{D_1 + D_2}{2} \right) \pi \alpha_c^* \\
& \left( h_3^2 + \left( \frac{D_1 - D_2}{2} \right)^2 \right)^{1/2}
\end{aligned}$$

= the sum of the areas of the shuttering

### 3.2.3 Constraints:

In seeking the optimum design which has the minimum value of the above formulated objective function, it has to satisfy certain requirements. These requirements turn out to be equations and/or inequalities which are functions of the design variables. These constraints are of two types. One type is geometric or side constraint which can be expressed explicitly in terms of design variables. The other type is based on the structural response called behaviour constraints and which cannot be normally expressed explicitly. The evaluation of the behaviour constraints requires the analysis of the structure at the current state.

The shuttering for conical dome is expensive and if the inclination angle is more than  $45^\circ$  one has to provide double shuttering, both on top and bottom of the dome. To

avoid this, the angle can be provided as less than or equal to  $45^\circ$ .

$$GJ(1) = \frac{h_3}{(D_1 - D_2)/2} = \tan \theta \leq 1$$

The ratio of the depth of breadth of the ring beam 3 is maintained between 1 and 3. This range of dimensions facilitates the direct access for the torsional coefficients during analysis and it also restricts the depth of the beam to increase to a large value.

$$1 \leq \frac{h_3}{b_3} \leq 4$$

This will come as two separate inequality constraints.

$$GJ(2) = 1.0 - \frac{h_3}{b_3} \leq 0, \quad GJ(3) = \frac{h_3}{b_3} - 4 \leq 0.$$

The minimum thickness of the conical shell and bottom thickness should not be less than 10 cm.

$$GJ(4) = 1.0 - \frac{t_4}{10} \leq 0; \quad GJ(5) = 1.0 - \frac{t_5}{10} \leq 0$$

The minimum thickness of the cylindrical shell at the bottom should be more than or equal to 15 cm. An upper bound can also be prescribed for this variable to avoid very large thickness of the wall.

$$GJ(6) = 15.0 - t_3 \leq 0; \quad GJ(7) = t_3 - 100.0 \leq 0$$

The thickness of the cylindrical wall at the bottom edge should be less than the breadth of the ring beam 2, which is an obvious requirement to avoid constructional difficulty. The depth of the ring beam should be more than the breadth of the beam.

$$GJ(8) = 1.0 - b_2/t_3 \leq 0, \quad GJ(9) = 1.0 - d_2/b_2 \leq 0$$

The geometry of the tank (container) should be such that it can store the given volume of water. This requirement can be expressed as a strict equality constraint in the optimization problem. But to avoid the difficulty which will arise in handling the equality constraint in the search procedure, the above requirement is expressed as two inequalities as follows.

The capacity should be more than(or equal to) the required volume but it should not be more than 5 percent of the required volume.

$$GJ(10) = 1.0 - \frac{\text{Volume(required)}}{\text{Capacity(provided)}} \leq 0$$

$$GJ(11) = \frac{\text{Volume required}}{\text{Capacity provided}} - 1.05 \leq 0$$

The reinforcements provided in various elements should not be less than 0.3 percent and also it should not exceed 2.3 percent of the cross section. This limitation creates

the following set of constraints.

$$GJ(12) = 0.003 - St_1 \leq 0, \quad GJ(13) = St_1 - 0.023 \leq 0$$

$$GJ(14) = 0.003 - St_2 \leq 0, \quad GJ(15) = St_2 - 0.023 \leq 0$$

$$GJ(16) = 0.003 - St_3 \leq 0, \quad GJ(17) = St_3 - 0.023 \leq 0$$

$$GJ(18) = 0.003 - St_4 \leq 0, \quad GJ(19) = St_4 - 0.023 \leq 0$$

$$GJ(20) = 0.003 - St_5 \leq 0, \quad GJ(21) = St_5 - 0.023 \leq 0$$

$$GJ(22) = 0.003 - St_6 \leq 0, \quad GJ(23) = St_6 - 0.023 \leq 0$$

where,  $St_1$  is reinforcement in the ring beam 1 ,

$St_2$  is reinforcement in the ring beam 2

$St_3$  is reinforcement in the bottom edge of the cylindrical wall

$St_4$  is reinforcement in the conical dome

$St_5$  is steel area in ring beam 3

$St_6$  is steel area of stirrups in the ring beam 3 per unit length, and they are fractions of the design variables.

$$GJ(24) = 0.003 - x_{12}, \quad GJ(25) = 0.023 - x_{12}$$

$$GJ(26) = 0.003 - x_{13}, \quad GJ(27) = 0.023 - x_{13}$$

$$GJ(28) = 0.003 - x_{14}, \quad GJ(29) = 0.023 - x_{14}$$

$$GJ(30) = 0.003 - x_{15}, \quad GJ(31) = 0.023 - x_{15}$$

where,  $x_{12}, x_{13}, x_{14}, x_{15}$  are design variables pertaining to steel area as detailed earlier.



The stresses in concrete and steel in the various members should be less than the allowable stresses. This limitation creates the following constraints. These constraints can not be expressed explicitly as function of design variables. However they can be evaluated by analysis procedure.

$$GJ(32) = \sigma_1 / \sigma_c - 1.0 \leq 0$$

$$GJ(33) = \sigma_2 / \sigma_t - 1.0 \leq 0$$

$$GJ(34) = \sigma_3 / \sigma_t - 1.0 \leq 0$$

$$GJ(35) = \sigma_6 / \sigma_c - 1.0 \leq 0$$

$$GJ(36) = \sigma_8 / \sigma_c - 1.0 \leq 0$$

$$GJ(37) = \frac{\sigma_9}{\sigma_{cb}} - 1.0 \leq 0$$

$$GJ(38) = \frac{\sigma_{10}}{\sigma_s} - 1.0 \leq 0$$

$$GJ(39) = \frac{\sigma_{11}}{\sigma_t} - 1.0 \leq 0$$

$$GJ(40) = \frac{\sigma_{12}}{\sigma_t} - 1.0 \leq 0$$

$$GJ(41) = \frac{\sigma_{13}}{\sigma_t} - 1.0 \leq 0$$

$$GJ(42) = \frac{(\sigma_{14} - \sigma_b)}{\sigma_t} - 1.0 \leq 0$$

$$GJ(43) = \frac{\sigma_{15}}{\sigma_{cb}} - 1.0$$

$$GJ(44) = \frac{\sigma_{16}}{\sigma_s} - 1.0$$

where ,  $\sigma_c$  - allowable direct compressive stress; 50 ksc for M200 and 40 ksc for M150,

$\sigma_t$  - allowable tensile stress in concrete; 17 ksc for M200,

$\sigma_s$  - allowable stress in steel; 1400 ksc,

$\sigma_{cb}$  - allowable compressive stress in bending; 70 ksc for M200 and 50 ksc for M150,

$\sigma_1$  - stress in concrete due to meridional thrust in the top dome,

$\sigma_2$  - stress in concrete due to hoop force in the top dome,

$\sigma_3$  - stress in concrete due to hoop force in the ring beam1,

$\sigma_6$  - stress in concrete in the conical dome,

$\sigma_8$  - stress in concrete due to meridional thrust in the bottom dome,

$\sigma_9$  - stress in concrete due to bending moment at the top dome edge

$\sigma_{10}$  - stress in steel due to bending moment at the top dome edge,

$\sigma_{11}$  - stress in concrete due to hoop force at the bottom edge of the cylindrical wall,

$\sigma_{12}$  - stress in concrete due to hoop force in the ring beam 2,

$\sigma_{13}$  - stress in concrete due to hoop force in the conical dome,

$\sigma_{14}$  - stress in concrete(tensile) due to bending moment at the edge of bottom dome,

$\sigma_{15}$  - stress in concrete due to bending moment at the bottom edge of conical dome,

$\sigma_{16}$  - stress in steel due to bending moment at the bottom edge of conical dome.

It can be seen that there are large number of constraints which arise due to the lower and upper bounds on the design variable. For example steel area should lie between 0.003 and 0.023. These kinds of constraints can be modified as equivalent constraints which eventually reduces the total number of constraints considerably.

Equivalent constraint for bounded variable constraint:

Suppose there are two constraints which relate the specified values and maximum and minimum of the design variable (or any function of design variables), then these two constraints can be converted into an equivalent constraint as follows.

$d_{\min} \leq d \leq d_{\max}$  can be expressed as  $d_{\min} \leq d$  and  $d \leq d_{\max}$  respectively.

Now,  $D_n = \frac{d_{\min} - d}{d_{\max} - d_{\min}} \leq 0$  is the normalised form of the first

constraint (i.e)  $D_n \leq 0$  and  $\frac{d - d_{\max}}{d_{\max} - d_{\min}} \leq 0$  is the normalised

form of the second constraint.

$$\frac{d - d_{\max}}{d_{\max} - d_{\min}} = \frac{d - d_{\min} + d_{\min} - d_{\max}}{d_{\max} - d_{\min}} = \frac{d - d_{\min}}{d_{\max} - d_{\min}} - 1 = -D_n - 1 \leq 0$$

In order to satisfy both the inequalities,  $D_n(-D_n - 1) \leq$  i.e.

$-D_n(D_n + 1) \leq 0$  has to be satisfied. If  $\frac{d - d_{\min}}{d_{\max} - d_{\min}} \geq 0$  is

considered as  $D_n$ , then the equivalent constraint becomes

$$D_n(D_n - 1) \leq 0.$$

The above mentioned method of modifying the constraints reduces the total number of constraints from 44 to 31 in the present problem.

### 3.3 Optimum Design Formulation for the Supporting Frame:

As mentioned earlier the supporting frame consists of columns and bracing beams which supports the container and transfers the load to the foundation. For a given capacity and height of staging one cannot predict easily the number of columns and number of panels which gives the minimum cost design of the frame. It is usual to provide equal panel height though there is no guarantee that the panels of equal height will give the minimum cost design of the frame. But in order to formulate the problem as a nonlinear programming problem one should predict the number of columns and number of panels before hand, otherwise it will have to be formulated as an integer non-linear programming problem with the panel

heights and number of columns and bracings as integer variables. Since the solution seeking of the nonlinear integer programming problem is not easy, the approach of parametric study has been adopted in the present work. In other words minimum cost design of the staging is studied for various sets of number of preassigned columns and braces. Based on the result of this study the number of columns and braces corresponding to the optimum design are given.

Suppose  $N_c$  and  $N_p$  are the number of columns and number of panels as the preassigned values for a given investigation. Then the formulation of the optimum design of staging proceeds as follows.

### 3.3.1 Design variables:

The dimensions of the bracing beams and columns and the reinforcements provided in these members, are the main variables which influences the cost of the supporting frame. Diameter of the column ( $d_c$ ) and area of steel ( $A_{st}$ ) in the column, breadth ( $b$ ), depth ( $d$ ) and the steel areas in the bracing beams are the variables in the problem. They are shown in Fig. 2. One can not consider all of them as design variables because the three variables in each column, 5 variables in each bracing and  $N_p$  variables of panel heights makes the design vector a large one. Even for a small value of  $N_p=3$  which is a practical

one, the size of design vector turns out to be 22. The design vector is reduced considerably by neglecting some variables after considering the following aspects.

1. The lateral ties in the columns and the stirrups in the beams are not considered as design variables because the effect of these variables over the cost is comparatively less than the effect of the other steel variables in the column and in the bracing beams.

2. It is not practicable to go in for the variation in the cross-sectional dimensions of the column along its height. Therefore, the diameter of the severe column is considered as the design variable and this dimension is kept same throughout the height of the column. Further, while the area of steel in different panels are considered as design variables, the area of steel within a panel is kept constant.

3. Since the number of panels is preassigned and the provision of equal panel height has been accepted in the present work, the variables regarding the heights of the panels are getting fixed automatically.

4. The bending moments in the various level of bracings are very nearly equal and they differ only by a slight quantity. Thus the dimensions, breadth and depth of the lowest level bracing beam can be considered as the design variables and the same dimensions can be provided for the other

bracing beams. However, to account for the variation in the bending moment at each bracing level, the area of steel in each bracing beam is considered as a design variable.

5. Depending on the wind direction the nature of the bending moment in the bracing beam changes. Hence it is conventional to provide equal steel area at the top and bottom of the bracing beam. This reduces one more steel variable in each bracing beam.

Thus the design vector consists of the following design variables.

$X_1$  - Diameter of the lower most panel column ( $d_c$ )

$X_2$  - breadth of the lower level bracing beam ( $b$ )

$X_3$  - depth of the lower level bracing beam ( $d$ )

$X_4, X_5 \dots X_{Np3}$  - steel areas in the columns of each panel ,

$Ast_1, Ast_2, Ast_3 \dots Ast_{Np}$  where  $Np3 = Np+3$

$X_{Np3+1}, X_{Np3+2}, \dots, X_{Np3+(Np-1)}$  - steel area in each bracing beam of various levels, (i.e.)  $Ast_{Np+1}, \dots, Ast_{Np+2}, Ast_{2Np-1}$

The variables corresponding to steel area are expressed as fraction of respective cross sections and  $(Np-1)$  denotes the total number of braces.

It is to be noted that while the number of columns does not affect the size of the design vector, the cost of the

Frame depends on the number of columns provided in the staging.

### 3.3.2 Objective function:

The objective function which represents the cost of the staging can be expressed in terms of the design variables defined above.

$$\text{Total cost of the frame } T_c = \text{Vol}_c \cdot C_c + W_s \cdot C_s$$

$$\text{where, volume of concrete } \text{Vol}_c = N_c \cdot H_t \frac{\pi}{4} d_c^2 + (N_p - 1) \cdot b \cdot d$$

$$(D \sin \frac{\pi}{N_c} - d_c) \cdot N_c$$

$$\text{Weight of steel in the columns } W_{sc} = \frac{\pi}{4} d_c^2 \frac{H_t}{N_p} (Ast_1 + Ast_2 + Ast_3$$

$$+ \dots \dots Ast_{N_p})^{N_c} + \pi d_c H_t \cdot N_c A_w / s)$$

$$\text{Weight of steel in the bracings } W_{sb} = \frac{\pi}{4} (bd(D \sin \frac{\pi}{N_c} - d_c) \cdot N_c \cdot$$

$$(2Ast_{(N_p+1)} + 2Ast_{(N_p+2)} \dots 2Ast_{(2N_p-1)} + \text{Stirrup steel}))$$

and  $W_s = W_{sc} + W_{sb}$  ;  $D$  = diameter of the ring beam ;  $H_t$  - height of staging;  $A_w$  - area of lateral ties;  $s$  - spacing of the lateral ties;  $C_c$  - cost of concrete per  $m^3$ ; and  $C_s$  - cost of steel per tonne.

### 3.3.3 Constraints:

The number of constraints of this problem depends on



the number of panels. Various geometric or side constraints and the behaviour constraints, on the problem, expressed as inequalities are explained below.

The minimum value of the diameter can be assessed depending on the breadth of the ring beam 3 and on the panel height in order to keep the column as a short column.

$$GJ(1) = 1.0 - X_1/b_{min} \leq 0$$

The breadth of the bracing should not be greater than the diameter of the column and from the practical consideration, it should also be slightly lesser than the diameter which may facilitate to extend the bars from the beams to the column in order to anchor it in the column reinforcements conveniently.

$$b \leq 0.8 d_c \text{ (i.e.) } GJ(2) = 1.0 - 0.8 X_1/X_2 \leq 0$$

The depth to breadth ratio should be within 1 and 4  $\frac{d}{b} \geq 1$  and  $\frac{d}{b} \leq 4$  are the two inequalities which can be expressed as one equivalent inequality as shown earlier.

$$GJ(3) = D_n(D_n - 1) \leq 0 \text{ where } D_n = \left(\frac{X_3}{X_2} - 1\right) / (4 - 1)$$

In each panel the columns are subjected to axial force and some bending moment. The safe design should satisfy the following inequality in each panel column for both wind and earthquake loading.

$$(i.e.) \quad \frac{\sigma_d}{\sigma_{ad}} + \frac{\sigma_b}{\sigma_{ab}} \leq 1$$

GJ(4), GJ(5), ...GJ(Np) are the constraints of the above form each one to the panel column and they can not be expressed explicitly by the design variables.

The areas of steel which are expressed as percentage of cross section should not be less than the minimum value of 0.008 and it should not be greater than 0.08(maximum)

$$GJ(Np+1) = D_n(D_n - 1.0) \leq 0 \text{ where } D_n = (X_4 - 0.008) / (0.08 - 0.008)$$

and GJ(Np+2), GJ(Np+3).....GJ(2Np-1) can also be expressed as above with the corresponding steel variables. Similar limitation on the steel variable of the bracing beam forms Np-1 number of constraints which are expressed as an equivalent inequality as shown below.

$$GJ(Np_2+1) = D_n(D_n - 1.0) \leq 0 \text{ where } D_n = (X_{3+Np} - 0.003) / (0.023 - 0.003)$$

and GJ(Np<sub>2</sub>+2), GJ(Np<sub>2</sub>+3), .....GJ(Np<sub>3</sub>) can also be expressed as above using the corresponding steel variables Np<sub>2</sub>=2Np-1, Np<sub>3</sub>=Np<sub>2</sub>+Np-1 and 0.003 and 0.023 are the minimum and maximum steel areas.

For example if the number of panels Np=4 , following fourteen constraints will be set up. First three for the limitations of the diameter of the column, breadth of the

bracing and the ratio of depth to breadth respectively. The next four constraints ( $N_p$  behaviour constraints) are for the stress limitations. The next four constraints ( $N_p$ ) for steel area limitations in the columns and the last three constraints ( $N_p-1$ ) for the steel area limitations in the three ( $N_p-1$ ) bracings. The total number of constraints are fourteen, Seventeen when  $N_p=5$ , and twenty when  $N_p=6$  irrespective of  $N_c$ .

### 3.4 Optimal Design Formulation for Foundation:

The type of the foundation is to be selected based on various considerations like the soil characteristics and the feasibility of construction etc. A ring beam-raft foundation is considered in this study. The size of this problem is comparatively smaller than the container and the staging design problem as is seen from the following sections.

#### 3.4.1 Design variables:

The foundation consists of a ring beam with raft slab and the geometric and steel area variables are shown in Fig.3. All possible variables are considered as design variables in this problem. The design vector consists of nine variables which are given below.

$X_1$ - breadth of the ring beam ( $b_1$ )

$X_2$ - depth of the ring beam ( $d_1$ )

- $X_3$ - thickness of the raft slab at the junction of the ring beam ( $t_1$ )  
 $X_4$ - thickness of the raft slab at the end ( $t_2$ )  
 $X_5$ - projection of the raft slab ( $d_2$ )  
 $X_6$ - area of steel at the top of the ring beam ,  $Ast_1$ , for negative B.M. at the support section  
 $X_7$ - area of steel at the bottom of the ring beam,  $Ast_2$  for positive B.M. at the middle section)  
 $X_8$ - area of steel at the support section of the slab-  $Ast_4$   
 $X_9$ - area of stirrup reinforcement for unit length of the beam,  $Ast_3$

$X_6$ ,  $X_7$ ,  $X_8$ ,  $X_9$  are the steel variables which are expressed as fraction of cross section of the respective elements.  $Ast_5$  which represents the distribution steel in the slab is not considered as design variable because it can be accounted as 0.15 percent of the cross section.

#### 3.4.2 Objective function:

The cost of the foundation can be expressed in terms of the design variables as given below.

Total cost of the foundation  $F = Vol_c C_c + W_s C_s$  where,  
 $Vol_c$ -volume of concrete  $= \pi D(x_1 x_2 + (2x_5 - x_1)(x_3 + x_4)/2.0)$

$W_s$  - weight of steel  $= \pi D(x_1 x_2 (x_6 + 0.7x_7) + x_1 0.6x_8 + x_5 (x_3 + x_4) (x_9 + 0.0015))$ ;

$\rho_s$  - density of steel

$D$  - diameter of the ring beam

$C_c, C_s$  - cost of concrete per  $m^3$ , cost of steel per tonne.

### 3.4.3 Constraints:

This foundation design problem consists of twenty inequality constraints out of which 13 are behaviour constraints and the rest are side constraints as detailed below.

The breadth of the beam should be greater than or equal to the diameter of the column ( $dia_c$ ).

$$b \geq dia_c ; \quad (i.e.) \quad GJ(1) = 1.0 - X_1/dia_c \leq 0$$

The ratio of breadth to depth of the ring beam should be between 1 and 4. These are  $\frac{d}{b} \leq 4$  and  $\frac{d}{b} \geq 1$  which can be represented as an equivalent constraint.

$$1 \leq \frac{d}{b} \leq 4 ; \quad GJ(2) = D_n(D_n - 1.0) \leq 0 \text{ where, } D_n = (\frac{x_2}{x_1} - 1.0)/3.0$$

The thickness of the raft slab at the end should not be less than the minimum thickness of 15 cm.

$$t_2 \geq 15 ; \quad GJ(3) = 1.0 - x_4/15.0 \leq 0$$

The projection of the raft slab should not be too large in which case it approaches to the complete raft slab type of

foundation and the analysis of the raft slab as a cantilever may not be valid.

$2d_2 \leq D-d_3$  ;  $GJ(4) = 2x_5/(D-d_3)-1.0 \leq 0$  where,  
 $d_3$  is the central diameter which checks the closing of the two raft slab projections.

The thickness of the slab  $t_1$  should be less than the depth of the ring beam

$$t_1 \leq d_1 ; \quad GJ(5) = 1.0 - x_3/x_4 \leq 0$$

The thickness of the raft slab at the end should be smaller or equal to that at the support end

$$t_1 \geq t_2 ; \quad GJ(6) = x_3/x_2 - 1.0 \leq 0$$

The actual bearing pressure should be within the safe allowable bearing pressure of the soil. This can be expressed in terms of the actual provided area and required area to be provided to keep the soil within the allowable limit.

$$\text{Actual area} \geq \text{Area required} ; \quad GJ(7) = 1.0 - \frac{\text{Area}_p}{\text{Area}_R}$$

The depth of the ring beam required to resist the bending moment as a singly reinforced section with the nominal cover of say five cms should be greater than (or equal to) the available depth  $d_1$  and similarly with the thickness of the raft slab  $t_1$ .

Depth required  $\geq d_1$  and thickness required  $\geq t_1$

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$$GJ(8) = 1.0 - \frac{X_5}{\text{depth}} ; \quad GJ(9) = 1.0 - \frac{X_3}{\text{thickness}}$$

The areas of steel expressed as fraction of cross section should be within the limits 0.003 and 0.023.

$$0.003 \leq Ast_1(X_6) \leq 0.023 ; \quad GJ(10) = D_{n1}(D_{n1}-1.0) \leq 0$$

$$0.003 \leq Ast_2(X_7) \leq 0.023 ; \quad GJ(11) = D_{n2}(D_{n2}-1.0) \leq 0$$

$$0.003 \leq Ast_4(X_8) \leq 0.023 ; \quad GJ(12) = D_{n3}(D_{n3}-1.0) \leq 0$$

$$0.003 \leq Ast_3(X_9) \leq 0.023 ; \quad GJ(13) = D_{n4}(D_{n4}-1.0) \leq 0$$

where,  $X_6, X_7, X_8, X_9$  are the steel variables as shown in the Fig.3,

$$\text{and } D_{ni} = (x_j - 0.003) / 0.020 \quad i=1,2,3,4$$

$$j=6,7,8,9$$

The stresses in concrete and steel should be within the allowable stress values. The following seven stress limit constraints are the behaviour constraints.

$$\sigma_1 \leq \sigma_s ; \quad \sigma_2 \leq \sigma_c ; \quad \sigma_3 \leq \sigma_s ; \quad \sigma_4 \leq \sigma_c$$

$$\sigma_5 \leq \sigma_s ; \quad \sigma_6 \leq \sigma_c ; \quad \sigma_7 \leq \sigma_{sh} ; \quad \text{where, } \sigma_s, \sigma_c,$$

$\sigma_{sh}$  are allowable stresses in steel, concrete in bending and in tension.

$\sigma_1$  - stress in steel due to the bending moment in the ring beam (support section)

$\sigma_2$  - stress in concrete due to the bending moment in the ring beam (support section)

- $\sigma_3$  - stress in steel due to the BM in the ring beam (midspan section)  
 $\sigma_4$  - stress in concrete due to the BM in the ring beam (midspan section)  
 $\sigma_5$  - stress in steel due to the BM at the raft slab support section  
 $\sigma_6$  - stress in concrete due to the B.M. at the raft slab support section  
 $\sigma_7$  - shear stress in concrete at the support section of the slab.

The above inequalities relating the actual and the allowable stresses can be put in the normalized form as follows.

$$GJ(14) = \frac{\sigma_1}{\sigma_s} - 1.0 \leq 0 \quad ; \quad GJ(15) = \frac{\sigma_2}{\sigma_c} - 1.0 \leq 0$$

$$GJ(16) = \frac{\sigma_3}{\sigma_s} - 1.0 \leq 0 \quad ; \quad GJ(17) = \frac{\sigma_4}{\sigma_c} - 1.0 \leq 0$$

$$GJ(18) = \frac{\sigma_5}{\sigma_s} - 1.0 \leq 0 \quad ; \quad GJ(19) = \frac{\sigma_6}{\sigma_c} - 1.0 \leq 0$$

$$GJ(20) = \frac{\sigma_7}{\sigma_{sh}} - 1.0 \leq 0$$

### 3.5 Summary:

The formulations of the optimization problems have been explained in this chapter. The design variables, the objective function and the constraints of each problem is discussed. Each one of them turns out to be a nonlinear programming problem. These are embedded in the development of the computer program which are explained in the subsequent chapter.



## CHAPTER 4

### OPTIMUM SEEKING METHOD AND PROGRAM DEVELOPMENT

#### 4.1 Introduction:

Once the problem of optimum design has been formulated, the solution of the problem can be obtained using efficient algorithms. The optimum design problems as explained in the last chapter are nonlinear programming problems. The method used for the solution of nonlinear programming problems and the details of the developed program are explained in this chapter.

#### 4.2 Solution Technique:

There are many algorithms available for the solution of nonlinear mathematical programming problem. Penalty function method is one among the category of indirect methods. The interior penalty function method is used in this work and the well known sequential unconstrained minimization technique (SUMT) is briefly described below.

##### 4.2.1 SUMT algorithm:

SUMT considers an unconstrained function  $\phi(X, r)$  where,  $r$  is the penalty parameter, as  $\phi(X, r) = f(x) - r \sum_{j=1}^m \frac{1}{g_j(x)}$ .  $f(x)$  is the objective function and  $g_j(x)$  are the constraints. As  $r$  tends to zero, the minimum of the objective function  $f(X)$

will be the minimum of the unconstrained function  $\phi(X, r)$ .

The unconstrained minimization is carried out by using

Davidon-Fletcher-Powell (DFP) variable metric algorithm in the present work. In this method, the search direction is

established as  $\vec{S}_i = - [H_i] \nabla \vec{f}_i$ , where  $[H_i]$  is the Hessian matrix and  $\nabla \vec{f}_i$  is the gradient vector  $(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n})^T$ .

Along the search direction  $\vec{S}_i$  the optimal step length  $\alpha_s^*$  at which the function is minimum in that linear direction is calculated using any one of the available one dimensional minimization methods. Fibonaeci method is used in this work which makes use of the sequence of Fibonacci numbers for placing the experiments in the interval of uncertainties. The number of experiments is decided as fifteen to get the accuracy of 0.00103. Having obtained the search direction,  $\vec{S}_i$  and step length,  $\alpha_s^*$ , we move to the new design point  $\vec{X}_{i+1}$  according to the expression,  $\vec{X}_{i+1} = \vec{X}_i + \alpha_s^* \vec{S}_i$ , where  $\vec{X}_i$  is the old design point. The Hessian matrix is updated in each iteration of unconstrained minimization as  $H_{i+1} = H_i + M_i + N_i$ , where ,

$$[M_i] = \frac{(\alpha_s^* \vec{S}_i)(\alpha_s^* \vec{S}_i)^T}{(\alpha_s^* \vec{S}_i)^T \vec{Q}_i} \quad ; \quad [N_i] = - \frac{[H_i] \vec{Q}_i \vec{Q}_i^T [H_i]}{\vec{Q}_i^T [H_i] \vec{Q}_i}$$

$$\text{and } \vec{Q}_i = \nabla \vec{f}_{i+1} - \nabla \vec{f}_i$$

The flow chart for the SUMT algorithm is shown in Fig.4.

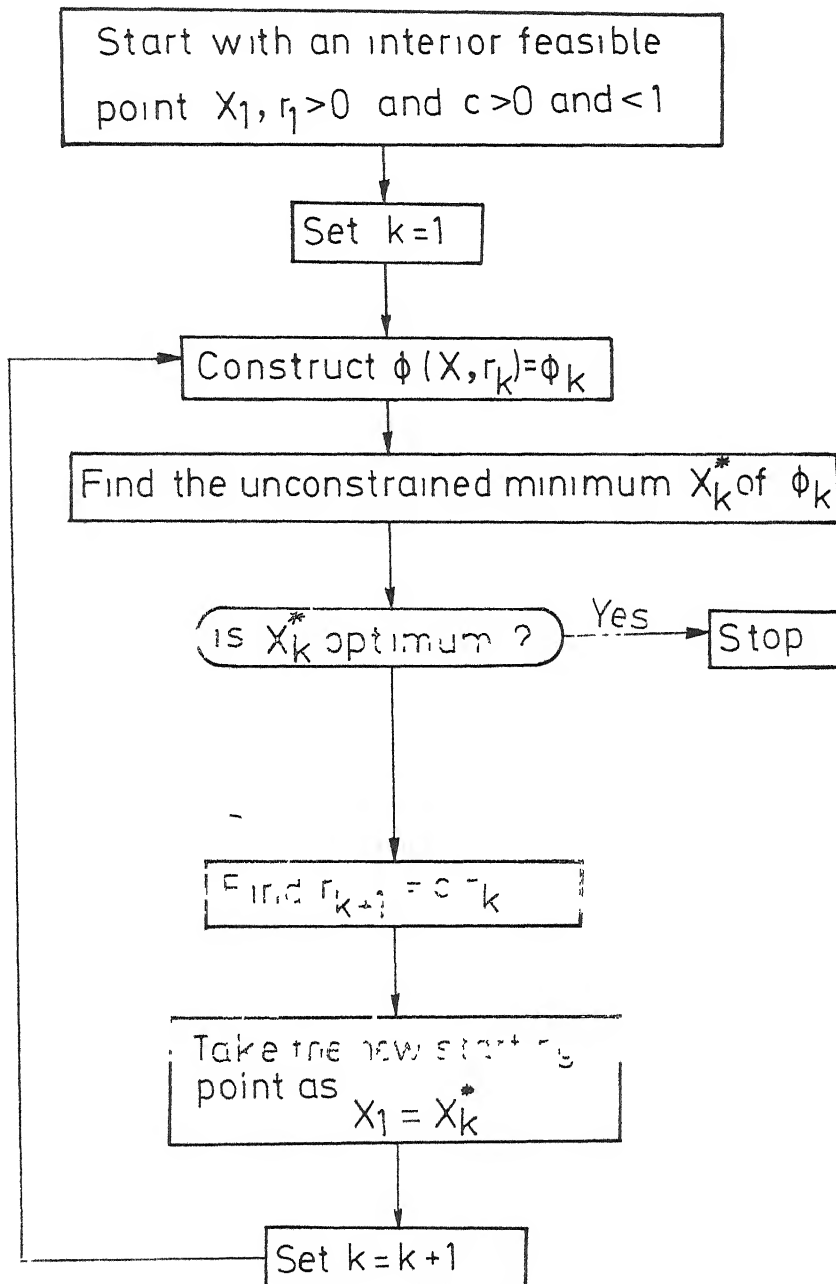


FIG.4 FLOW CHART FOR SUMT ALGORITHM

In order to have an efficient algorithm for linear minimization to obtain,  $\alpha_s^*$ , the initial interval of uncertainty for bracketing the minimum along that direction is to be very carefully fixed. The following procedure has been used to automatically fix up the initial interval of uncertainty in the present work.

1. Find the values of  $\alpha_1, \alpha_2, \dots, \alpha_{jk}$  where,  $\alpha_i = \frac{x_i}{|s_i|}$  and  $jk$  is the number of negative elements in the vector of search direction. These values represent respectively the limiting values of step length to avoid the non-negativity of the corresponding variable in the next design point.
2. Select the minimum value among  $\alpha_1, \alpha_2, \dots, \alpha_{jk}$  and check whether the design point with this minimum value of this step length is feasible or not. Set this minimum  $\alpha$  as  $\alpha_{\min}$ .
3. If the design point is feasible then the initial interval of uncertainty is  $A_1-B_1$  where  $A_1=0.0$  and  $B_1=\alpha_{\min}$ . If it is not feasible, reduce it to  $\alpha = c.\alpha$  where  $c$  is the reduction factor and check for the violation of constraints and repeat the process till the value  $\alpha_{\min}$  is obtained which satisfies non-negativity of design variables and the constraints.
4. In some cases there may be a search direction which has no negative element. In such a situation any interval of uncertainty apriori can be fixed.

5. In the reduction process, once the feasible region approaches zero or a very small quantity then  $\alpha$  is set to zero.

The above strategy to select the initial interval of uncertainty is described in the flow chart given in Fig.5.

#### 4.3 Program Development:

Any structural optimization program , particularly nonlinear programming problems,basically consists of the following main parts.

1. Routine which evaluates the value of the objective function
2. Routine for carrying out the analysis of the structure
3. Routine for checking of the constraints.
4. Routine for the solution technique to reach the optimum.

In this work, three separate computer programs are developed which have similar format except for the difference in the corresponding methods of analysis and their objective function and constraints.

Following are some situations in which the programs need certain modifications. When the balcony slab is provided at the ring beam 2 level, the corresponding moment and forces are to be considered in the analysis and the necessary modifications can be made accordingly. When the cross-sections

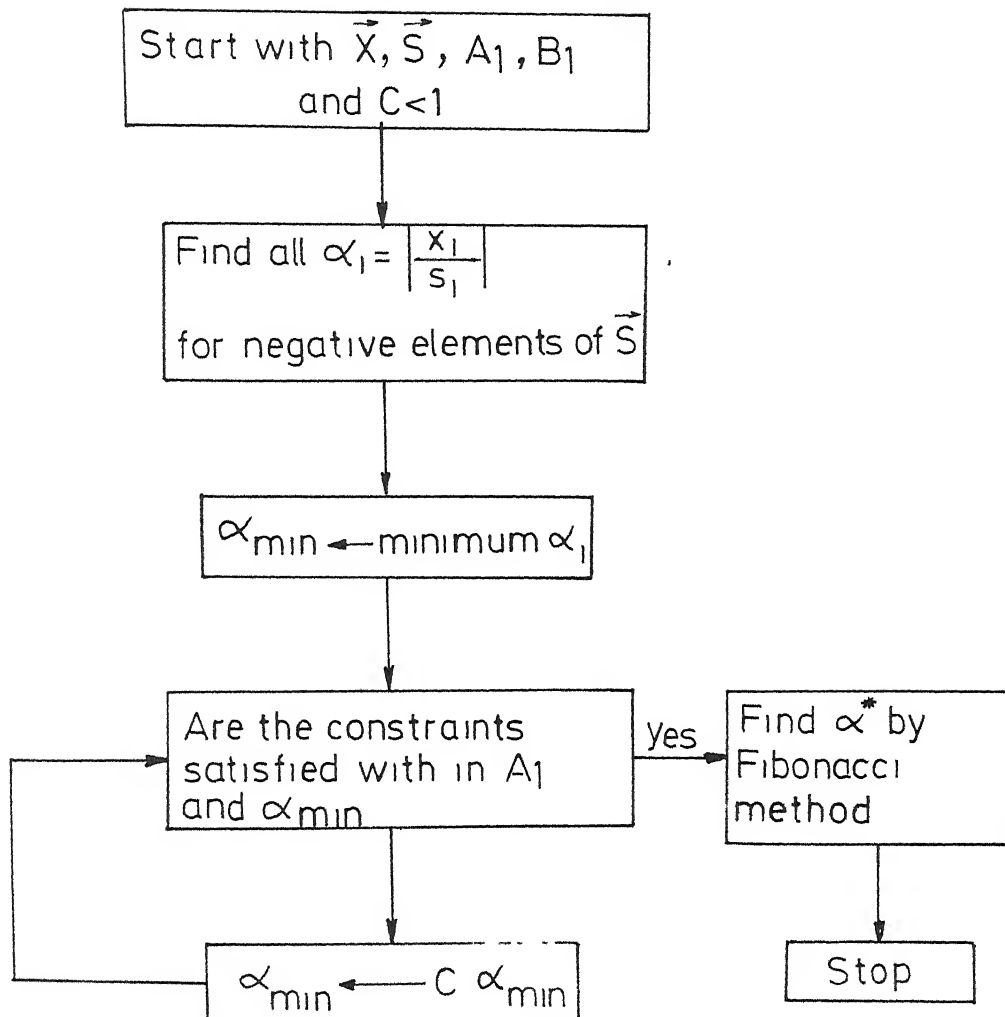


FIG.5. FLOW CHART TO FIND INITIAL INTERVAL OF UNCERTAINTY

columns are required as square or rectangular the design vector and the necessary changes in the analysis can be easily done. For certain combinations of the number of columns and bracings the present analysis of the staging may turn out to give crude values. For analysis of the staging with less number of columns, say four, and/or for less number of bracings, the present analysis should not be used.

Program 1 : Optimal design of the container:

This program consists of number of routines which are explained below:

Subroutine MAIN: This contains all the necessary data for the design and analysis of the container. The values of (1) the initial design point, (2) the maximum and minimum values of the design variables for normalization, (3) the cost details of concrete, steel and shuttering, (4) the allowable stress values of concrete and steel (5) the conical shell coefficients which are required to evaluate stiffness (6) other preassigned parameters and capacity, number of columns are all given in DATA cards. They are transferred to the required subroutines through COMMON statements. Apart from setting up and transferring the necessary values to other routines, this main calls SUMT (algorithm) routine with the initial design point which returns the optimal design point.

Subroutine SUMT: This routine forms the unconstrained function ( $\phi$ ) using the objective function, constraints and the appropriate penalty parameter and calls DFP to get a new design point. The penalty parameter  $r$  is reduced in each iteration and the corresponding unconstrained function is minimized using DFP. It returns the optimal design point to the main program.

Subroutine DFP: This routine calls the other routines, DIFF, SEARCH, FUNC, FIBBN, HEASIAN. This routine essentially does the calculations as shown in Fig.4. With the starting Hessian matrix which is a unit matrix and the gradient vector at the starting point (using DIFF) it calls SEARCH and obtains the optimum step length from FIBBN. It returns the optimum design point to SUMT once the convergence criteria is satisfied.

Subroutine DIFF: This routine calculates the gradient vector using the central difference formula. This calls FUNC to evaluate the function values at the forward and backward point for each design variable. The probe length EPQ is taken as 1 percent of the design variable value ( $EPQ=0.01$ ) . EPQ can be altered if the higher or lower probe length is derived.

Subroutine SEARCH: This routine returns the normalized search direction vector  $\vec{S}$ . The Hessian matrix and the gradient vector  $(-\vec{\nabla}f)$  are multiplied in this subroutine and is added with the previous search direction to get the new search direction.



Subroutine HESIAN: This routine does the necessary calculations for updating the Hessian matrix. The required quantities for updating the matrix are the current Hessian matrix  $[H_i]$ , the difference of gradient vector ( $\vec{Q}_i$ ), search direction vector ( $\vec{S}_i$ ) and the optimum step length ( $\alpha_s^*$ ). The additional matrices  $[M_i]$  and  $[N_i]$  are generated and added with  $[H_i]$  updated Hessian Matrix  $[H_{i+1}]$  is returned to the routine DFP.

Subroutine FIBBN: This routine returns the optimum step length  $\alpha_s^*$  in the search direction. This routine calls PHIF which evaluates the function value at any point along the search direction. This routine implements the strategy to select the proper initial interval of uncertainty as explained earlier, and then searches for  $\alpha_s^*$  using the fibonacci search technique.

Subroutine PHIF: This routine returns the function value ( $\phi$ ) at any point ALFA along the search direction  $\vec{S}$  from the given point  $\vec{X}_1$ . This calls FUNC to evaluate the value and sends the value to FIBBN.

Subroutine FUNC: This routine evaluates the unconstrained function value ( $\phi$ ). It calls the routine FN to evaluate the objective function value and calls CONSUE to get the summation of the inverse values of the constraints. The necessary penalty parameter is transferred via COMMON from SUMT.

Subroutine FN : This routine returns the objective function value for any given design point. It calculates the concrete and steel quantities and shuttering area of the various elements and using the corresponding cost values it evaluates the objection function. The necessary invariant parameters are transferred through COMMON statement.

Subroutine CONSUE: This routine contains the geometric constraints and the behaviour constraints and it returns the constraint values and the summation of the inverse values of the constraints to other routines. It calls ANALYS to get the values for some quantities in the behaviour constraint expressions.  $GJ(n)$  represents the  $n$ th constraints and  $GJX$  represents the summation.

Subroutine ANALYS: This routine is developed as a separate program for the analysis of the container of the intze tank. Both the membrane analysis and continuity analysis are carried out in this routine. For a given set of values of the variables and other assigned parameters this routine gives the forces and moments and the resulting stresses and the corresponding reinforcement areas required. The values of stresses, steel areas, and volume are returned to CONSUE from this routine .

Subroutine SOLVE: This routine returns the solution of the system of equations giving thereby values of displacement and rotation (DISP and ROT) corresponding to the compatibility requirements. This is called thrice by ANALYS for each set of rotation and displacement at the three ring beam junction. LU factorization technique with pivoting strategy is implemented, in order to avoid numerical instability.

Subroutine CURBCO: This routine returns the coefficients  $\beta_1, \beta_2$  and  $\beta_3$  to calculate the design moments in a circular beam which is supported by NSUP or SUPN number of columns. Since the coefficients are dependent only on the number of columns which is a preassigned parameter, this is called only once by the main program and transferred to ANALYS via COMMON.

Program 2: This program for the optimal design of the staging has the similar routines performing similar operations. The main program, analysis routine, routine for calculating the value of the objective function and constraints are all corresponding to the problem of the supporting frame, whereas the routines for SUMT algorithm remains the same.

Subroutine MAIN: This routine has all the values required for the other routines. The initial design point vector, maximum and minimum limits for normalization, other preassigned parameter like the height of staging, number of columns and panels are

all available in DATA cards and are transferred to the required routines via COMMON statements. The values required for the resultant wind force and the distance at which it acts, are all given as input which are derived from the optimal design of the container (from program 1).

The other subroutines from SUMT to PHIF are exactly the same in this program.

Subroutine FN: This routine calculates the concrete and steel quantities in the columns and bracing beams and then it evaluates the value of the objective function using the costs of concrete and steel and returns this to FUNC.

Subroutine CONSUE: This routine evaluates the values of all the constraints(given in the earlier chapter)and also the summation of the inverse of the constraints. The number of constraints depends on the number of panels NP.

Subroutine ANALYS: This routine performs the analysis of the supporting frame both for wind force and earthquake loading. The bending moment and twisting moment which gives maximum values for earthquake or wind force are considered as design forces'. Similarly the sum of stress ratios is calculated for both wind and earthquake cases, and the maximum one is returned to CONSUE.

Program 3: This program which also has similar routines as in the above two programs is to find the optimum design point for the foundation.

MAIN : This has the values of the loads that are to be transferred to the soil, the bearing capacity of the soil, and the initial design point and the limiting values of the variables for normalisation. It sends the values of the curved beam coefficients to ANALYS which are obtained by calling CURBCO.

Subroutine FN: This calculates the objective function value i.e. the cost of the foundation at any design point by evaluating the concrete and steel quantities and using the respective cost values.

Subroutine ANALYS: It performs the analysis of the ring beam and raft slab and calculates the required area of steel and the corresponding stresses in concrete and steel at the points of maximum force or moment. It requires the curved beam coefficients which are transferred from MAIN program.

## CHAPTER 5

### RESULTS, DISCUSSION AND CONCLUSION

#### 5.1 Introduction:

Two problems as detailed below respectively of 600 KL and 1000 KL capacity have been solved to obtain the minimum cost design of R C Intze tank by the use of the developed programs. All the results given have been obtained on the IBM 7044/1401 computer at IIT Kanpur. The bearing capacity of the soil has been taken to be  $20 \text{ t/m}^2$  in each case. The wind pressure on the structures is assumed to be  $150 \text{ kg/m}^2$ . All the allowable stresses considered in the present work are as per IS Code.

#### 5.2 Details of the Problems:

Problem 1: Capacity of the tank : 600 KL

Height of Staging : 15 M

Concrete Used : M200

Problem 2: Capacity of the tank : 1000 KL

Height of Staging : 22 M

Concrete used for : M200  
Container and Foundation

Concrete used for : M250  
Staging

The parametric study has been carried out by changing the number of columns and the number of panels in the staging. For 600 KL capacity towers, the number of columns considered in the present work are 6, 8 and 10 and the number of panels for its 15 m staging have been taken as 3, 4 and 5 in each case. For 1000 KL capacity tower, the number of columns considered are 8, 10, and 12 and the number of panels for its 22m staging are 4, 5 and 6 in each case.

For brevity, these problems are referred or designated as follows:

C(600,6) means the container problem of 600 KL capacity with 6 columns.

S(600,6,4) means the staging with 6 columns and 4 panels supporting the 600 KL capacity container.

F(600,6,4) means the foundation for the 600 KL capacity container tank with the staging contains 6 columns and 4 panels.

### 5.3 Results:

The minimum cost of the water tank is obtained using the programs 1, 2 and 3 in an interactive manner. That is, for a given capacity, the optimal design of the container is obtained first, using program 1. Then program 2 is energised where the loads coming on each column are calculated using the

TABLE 2: C(1000,10): CONTAINER OPTIMIZATION RESULTS

$\bar{x}$	Starting $\bar{x}$	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	$\bar{x}_4$	$\bar{x}_5$
$x_1$	13.75m	13.66	13.66	13.65	13.65	13.65
$x_2$	6.00m	5.99	5.99	6.00	6.00	6.00
$x_3$	35.00 cm	41.80	41.80	31.50	31.50	31.50
$x_4$	60.00 cm	62.50	62.50	60.34	60.33	60.33
$x_5$	85.00 cm	85.60	85.60	84.16	84.15	84.15
$x_6$	65.00 cm	69.20	69.20	62.87	62.85	62.85
$x_7$	1.70 m	1.70	1.703	1.708	1.707	1.707
$x_8$	10.00 m	10.00	10.00	10.03	10.03	10.03
$x_9$	60.00 cm	49.20	49.2	46.5	46.52	46.52
$x_{10}$	100.00 cm	98.80	98.80	97.86	97.96	97.96
$x_{11}$	20.00 cm	20.90	20.90	14.57	14.57	14.57
$x_{12}$	0.004	0.021	0.0068	0.0033	0.0031	0.0031
$x_{13}$	0.004	0.014	0.004	0.0032	0.003	0.003
$x_{14}$	0.004	0.018	0.0038	0.0032	0.0031	0.0031
$x_{15}$	0.01	0.004	0.0039	0.0037	0.0031	0.0031
$\nu$	-	402	40.2	4.02	0.4	0.04
$E_N$	118316.67	165227.8	1222802.3	104592.1	103705.1	103705.1
<hr/>						
Active	1, 11, 12, 13, 22, 23, 24	8, 11, 14, 1 31	8, 12, 13, 14, 1, 31	8, 11, 12, 12, 8, 13, 14, 22, 23, 24, 31	12, 8, 11, 11, 13, 14 22, 23, 24 1, 31	12, 8, 11, 13, 14, 22, 23, 24, 1, 31
Constraints	31					



TABLE 3: PROBLEM S(600,10,4): OPTIMIZATION THE STAGING

Design Vector $\vec{x}$	Starting $\vec{x}$	$\vec{x}_1$	$\vec{x}_2$	$\vec{x}_3$	$\vec{x}_4 = \vec{x}^*$
$x_1$	50.00 cm	49.89	49.89	49.8	49.8
$x_2$	25.00 cm	24.90	24.90	24.90	24.9
$x_3$	60.00 cm	59.90	59.90	59.90	59.90
$x_4$	0.01	0.015	0.00818	0.0089	0.0086
$x_5$	0.01	0.0085	0.035	0.0083	0.0081
$x_6$	0.01	0.014	0.0108	0.00812	0.0084
$x_7$	0.01	0.015	0.0818	0.0818	0.008
$x_8$	0.015	0.0053	0.0037	0.00313	0.003
$x_9$	0.015	0.0053	0.0035	0.00307	0.00314
$x_{10}$	0.015	0.0053	0.0034	0.0033	0.00312
$r$	-	64.37	6.43	0.643	0.064
$F_N$ (Rs.)	25351.33	23298.53	23821.08	18965.91	18875.59
Active Const- raints	5,6,8,9, 10,11,	9,5,6,8, 10,11	9,5,6,8,10, 11,12,13, 14	9,5,10,11, 12,13,14, 6,8	9,5,10,11,12, 13,14,6,8

$r$ - penalty parameter

TABLE 4: PROBLEM F(600, 8, 4): FOUNDATION OPTIMIZATION

Design Vector	Starting $\vec{x}$	$\vec{x}_1$	$\vec{x}_2$	$\vec{x}_3 = \vec{x}^*$
$x_1$	52.0 cm	53.67	53.4	53.4
$x_2$	90.0 cm	54.49	57.09	55.09
$x_3$	25.0 cm	29.53	29.51	29.50
$x_4$	20.0 cm	17.08	17.03	17.00
$x_5$	100.0 cm	94.92	94.32	94.28
$x_6$	0.01	0.0161	0.011	0.0109
$x_7$	0.005	0.0103	0.0055	0.0053
$x_8$	0.01	0.0174	0.015	0.004
$x_9$	0.008	0.0065	0.0063	0.0062
$r$	-	19.48	1.948	0.194
$F_N$ (Rs.)	12433.58	10941.08	9444.08	9420.88
Active Constraints	1, 6, 7, 18, 19	1, 2, 7, 15, 18, 19	1, 2, 7, 11, 14, 15, 16, 18, 19	1, 2, 7, 11, 14, 15, 16, 18, 19

TABLE 5 : CONTAINER OPTIMIZATION RESULTS

Design Vector	C(600,6)		C(600,8)		C(600,10)	
	Starting Point	Optimum Point	Starting Point	Optimum Point	Starting Point	Optimum Point
$X_1(m)$	12.25	12.18	12.25	12.24	12.25	12.24
$X_2(m)$	4.50	4.47	4.50	4.43	4.50	4.43
$X_3(cm)$	25.00	27.52	25.00	28.09	25.00	28.07
$X_4(cm)$	40.00	29.59	40.00	38.89	40.00	38.80
$X_5(cm)$	45.00	56.80	45.00	51.63	45.00	51.60
$X_6(cm)$	45.00	45.15	45.00	47.96	45.00	47.90
$X_7(m)$	1.50	1.42	1.50	1.426	1.50	1.426
$X_8(m)$	9.00	8.96	9.00	8.975	9.00	8.98
$X_9(cm)$	45.00	20.02	45.00	20.21	45.00	20.20
$X_{10}(cm)$	80.00	76.99	80.00	74.28	80.00	74.28
$X_{11}(cm)$	20.00	15.06	20.00	11.83	20.00	11.80
$X_{12}$	0.004	0.0152	0.004	0.0032	0.004	0.0036
$X_{13}$	0.004	0.0170	0.004	0.0038	0.004	0.0038
$X_{14}$	0.004	0.004	0.004	0.003	0.004	0.003
$X_{15}$	0.02	0.0072	0.01	0.004	0.01	0.0038
$r_1$	350.00	-	273.00	-	273.80	-
$r^*$	-	0.350	-	0.273	-	0.273
$F_N(Rs.)$	71336.7	70721.09	68227.78	62298.67	68227.78	62329.67

Table 5 contd. ....

Table 5 contd.....

Design Vector $\vec{x}$	C(1000, 8)		C(1000, 12)	
	Starting Point	Optimum Point	Starting Point	Optimum Point
$X_1(m)$	13.75	13.65	13.75	13.68
$X_2(m)$	6.00	6.00	6.00	5.97
$X_3(cm)$	35.00	31.52	35.00	43.28
$X_4(cm)$	60.00	60.33	60.00	62.12
$X_5(cm)$	85.00	84.15	85.00	85.50
$X_6(cm)$	65.00	62.85	65.00	69.74
$X_7(m)$	1.70	1.707	1.70	1.69
$X_8(m)$	10.00	10.03	10.00	9.98
$X_9(cm)$	60.00	46.52	60.00	35.66
$X_{10}(cm)$	100.00	97.86	100.00	96.50
$X_{11}(cm)$	20.00	14.54	20.00	23.39
$X_{12}$	0.004	0.016	0.004	0.0039
$X_{13}$	0.004	0.0169	0.004	0.00301
$X_{14}$	0.004	0.0039	0.004	0.00306
$X_{15}$	0.02	0.0076	0.01	0.0031
$r_1$	402.00	-	282.00	-
$r^*$	-	0.40	-	0.28
Function Values (Rs.)	126058.52	123987.60	118265.32	117822.00

$r_1$  - Initial penalty parameter;  $r^*$  - Final penalty parameter

TABLE 6 : STAGING OPTIMIZATION RESULTS

Design Vector	S(600,6,3)		S(600,6,4)		S(600,8,4)	
	Starting Point	Optimum Point	Starting Point	Optimum Point	Starting Point	Optimum Point
$X_1$ (cm)	60.00	63.45	60.00	64.36	55.00	53.30
$X_2$ (cm)	25.00	24.45	25.00	24.60	25.00	24.39
$X_3$ (cm)	60.00	41.90	60.00	59.80	60.00	59.70
$X_4$	0.02	0.00838	0.02	0.0082	0.02	0.0079
$X_5$	0.02	0.0087	0.02	0.00827	0.02	0.0087
$X_6$	0.02	0.00802	0.02	0.00849	0.02	0.0081
$X_7$	0.015	0.0039	0.02	0.008	0.02	0.009
$X_8$	0.015	0.0030	0.15	0.00303	0.015	0.0031
$X_9$	-	-	0.015	0.00303	0.015	0.0031
$X_{10}$	-	-	0.015	0.00302	0.015	0.0032
$X_{11}$	-	-	-	-	-	-
$X_{12}$	-	-	-	-	-	-
$r_1$	344.1	-	210.00	-	347.60	-
$r^*$	-	0.344	-	0.210	-	0.347
$F_N$ (Rs.)	25624.3	16054.8	28903.0	18535.90	31169.6	17630.4

Table 6 contd.....

Table 6 contd.....

Design Vector $\bar{x}$	S(600,8,5)		S(600,10,3)		S(600,10,5)	
	Starting Point	Optimum Point	Starting Point	Optimum Point	Starting Point	Optimum Point
$X_1$ (cm)	55.00	54.98	55.00	47.90	55.00	54.60
$X_2$ (cm)	25.00	24.89	25.00	20.32	25.00	24.80
$X_3$ (cm)	60.00	59.90	60.00	57.96	57.96	59.90
$X_4$	0.02	0.008	0.02	0.00804	0.02	0.008
$X_5$	0.02	0.0083	0.02	0.0081	0.02	0.008
$X_6$	0.02	0.008	0.02	0.0087	0.02	0.008
$X_7$	0.02	0.0082	0.015	0.0034	0.02	0.008
$X_8$	0.02	0.0080	0.015	0.0032	0.02	0.008
$X_9$	0.015	0.0031	-	-	0.015	0.003
$X_{10}$	0.015	0.0035	-	-	0.015	0.032
$X_{11}$	0.015	0.0035	-	-	0.015	0.0050
$X_{12}$	0.015	0.0036	-	-	0.015	0.0033
$r_1$	157.9	-	636.9	-	437.50	-
$r^*$	-	0.157	-	0.0636	-	0.0436
$F_N$ (Rs.)	34414.0	19930.0	33046.52	15820.51	39298.20	23247.69

TABLE 7 : STAGING OPTIMIZATION RESULTS

Design Vector	S(1000,8,4)		S(1000,8,6)		S(1000,10,6)	
	Starting Point	Optimum Point	Starting Point	Optimum Point	Starting Point	Optimum Point
$X_1$ (cm)	60.00	54.80	60.00	53.30	60.00	56.00
$X_2$ (cm)	25.00	21.20	25.00	21.50	25.00	23.70
$X_3$ (cm)	60.00	57.60	60.00	58.38	60.00	59.40
$X_4$	0.03	0.03	0.03	0.0083	0.03	0.0084
$X_5$	0.03	0.008	0.03	0.057	0.03	0.016
$X_6$	0.03	0.008	0.03	0.0086	0.03	0.011
$X_7$	0.03	0.0084	0.03	0.0082	0.03	0.0088
$X_8$	0.015	0.0033	0.03	0.008	0.03	0.0083
$X_9$	0.015	0.003	0.03	0.0081	0.03	0.0084
$X_{10}$	0.015	0.003	0.015	0.0031	0.015	0.0035
$X_{11}$	-	-	0.015	0.013	0.015	0.003
$X_{12}$	-	-	0.015	0.015	0.15	0.0031
$X_{13}$	-	-	0.015	0.0035	0.015	0.003
$X_{14}$	-	-	0.015	0.010	0.015	0.0033
$r_1$	108.2	-	79.2	-	102.0	-
$r^*$	-	0.01	-	0.079	-	0.01
$F_N$ (Rs.)	60415.6	30820.0	67878.92	39273.81	79528.1	38443.97

Table 7 contd.....

Table 7 contd...

Design Vector $\vec{x}$	S(1000, 12, 4)		S(1000, 12, 5)	
	Starting Point	Optimum Point	Starting Point	Optimum Point
$X_1$ (cm)	60.00	58.00	60.00	59.00
$X_2$ (cm)	25.00	24.70	25.00	24.80
$X_3$ (cm)	60.00	59.50	60.00	59.90
$X_4$	0.03	0.0081	0.03	0.0085
$X_5$	0.03	0.009	0.03	0.01
$X_6$	0.03	0.009	0.03	0.008
$X_7$	0.03	0.008	0.03	0.0083
$X_8$	0.015	0.0048	0.03	0.00817
$X_9$	0.015	0.0044	0.015	0.0138
$X_{10}$	0.015	0.02	0.015	0.012
$X_{11}$	-	-	0.015	0.0056
$X_{12}$	-	-	0.015	0.0065
$X_{13}$	-	-	-	-
$X_{14}$	-	-	-	-
$r_1$	165.70	-	139.0	-
$r^*$	-	0.165	-	0.139
$F_N$ (lb.)	84182.17	44976.1	87632.5	46320.40



TABLE 8 : FOUNDATION OPTIMIZATION RESULTS

Design Vector $\bar{x}$	F(600,6,3)		F(600,6,4)		F(600,8,5)	
	Starting Point	Optimum Point	Starting Point	Optimum Point	Starting Point	Optimum Point
$x_1$ (cm)	65.00	77.40	65.00	69.32	55.00	55.90
$x_2$ (cm)	100.00	93.36	100.00	77.88	90.00	70.40
$x_3$ (cm)	30.00	26.96	30.00	38.18	25.00	20.00
$x_4$ (cm)	25.00	16.01	25.00	15.00	20.00	14.99
$x_5$ (cm)	150.00	143.00	150.00	145.05	100.00	84.62
$x_6$	0.02	0.0097	0.02	0.011	0.01	0.0032
$x_7$	0.01	0.0049	0.01	0.0056	0.005	0.003
$x_8$	0.02	0.013	0.01	0.0043	0.01	0.0166
$x_9$	0.01	0.0084	0.01	0.0062	0.008	0.003
$r_1$	44.80	-	42.4	-	21.59	-
$r^*$	-	0.44	-	0.004	-	0.002
$F_N$ (lb.)	26259.2	18297.17	26235.1	16987.0	12808.35	6232.1

Table 8 contd.....

Table 8 contd...

Design Vector $\bar{x}$	F(1000, 8, 6)		F(1000, 10, 6)		F(1000, 12, 6)	
	Starting Point	Optimum Point	Starting Point	Optimum Point	Starting Point	Optimum Point
$X_1(\text{cm})$	55.0	63.90	58.00	67.90	62.00	67.36
$X_2(\text{cm})$	100.0	92.08	90.00	84.63	90.00	82.98
$X_3(\text{cm})$	50.0	38.90	50.00	43.36	50.00	36.38
$X_4(\text{cm})$	25.0	18.51	25.00	17.28	25.00	19.66
$X_5(\text{cm})$	200.0	152.48	200.00	180.94	200.00	156.93
$X_6$	0.02	0.01	0.02	0.0086	0.02	0.0075
$X_7$	0.01	0.0052	0.01	0.00434	0.01	0.0037
$X_8$	0.022	0.0093	0.022	0.005	0.022	0.01
$X_9$	0.01	0.0082	0.01	0.008	0.01	0.009
$r_1$	65.82	-	64.07	-	66.37	-
$r^*$	-	0.658	-	0.064	-	0.066
$F_N(R, )$	37417.69	22070.0	36579.5	24092.6	37424.0	20624.4

the geometry of the optimised container, and wind pressure and earthquake requirements are given as data. This program then obtains the optimal design of the staging. Finally, the data from the program 2, like the load coming on the foundation, the diameter of the bottom ring beam etc. are used in program 3 so as to obtain the optimal foundation design.

#### 5.4 Discussion:

The problems which have been solved in the preceding section show the flexibility of the developed programs which can be used for any capacity, any height of staging with a given number of columns and panels to obtain the optimal design.

The starting point of the problem C(600,8) (i.e.) container of 600 KL capacity supported on 8 columns and the optimal design point of the same problem are given in the Table 5. The cost of the container is minimized from Rs. 68227 to Rs. 62298 which shows 8.69 percent reduction. The starting point for this case is the design vector worked out in Ref.(8).

The minimization history shown for the problem C(1000,10) shows that the geometric variables (i.e.) diameter of the cylindrical wall,  $X_1$ , height of the cylindrical wall,  $X_2$ , height of the conical shell,  $X_7$ , and the diameter of the ring beam 3,  $X_8$ , are not significantly different from

the starting point and the optimum point. This is because of the constraint GJ(8). Since this constraint is a function of the variables which affects the volume of the container, any small change in these variables which may reduce the function value is prevented in order to satisfy this constraint. This observation leads us to conclude that the size of the problem can be further reduced by fixing the configuration (neglecting  $X_1$ ,  $X_8$ ,  $X_2$ ,  $X_7$  as design variables) of the container in which case the shuttering cost shall be constant and need not be considered as a part of the optimal design formulation.

It can be seen that the optimal cost of the container for a given capacity varies with the variation of the number of columns in the staging. The optimal cost of the container for 600 KL capacity is more for 6 number of columns as compared to that of 8 and 10 columns. Similar observation is made for 1000 KL capacity also. That is, the cost for 8 columns is more as compared to that of 10 and 12 columns. This is because the bending moment in the ring beam 3 is more when the number of columns is less.

The optimal design of the staging for problems are shown in the Table 7 . It can be seen that the minimization is achieved by the reduction of the steel in the columns and bracing beams towards the minimum reinforcement requirement

as per IS code and diameter of the column is getting fixed accordingly. In the staging where the number of columns are lesser say  $S(600,6,4)$ , the diameter of the column becomes larger at the optimum design corresponding to minimum reinforcement, in order to carry the load coming on it. It is obvious that the load coming on a column is reduced when the staging contains more number of columns. However, it can be seen from the Table 7 that the minimum cost of the staging are not significantly different as the number of columns vary. But the cost of the foundation varies significantly for the variation in the number of columns because the cost of the foundation depends on the load coming on the column (load per span on the bottom ring beam between the two columns) and also on the self weight of the staging. For 600 KL capacity problem the cost of staging for  $S(600,6,4)$ ,  $S(600,8,5)$  and  $S(600,10,4)$  are around Rs. 19,000 but the foundation cost of the corresponding problems show significant variation for the obvious reason explained above. Thus it can be concluded that the optimum design of the staging can be obtained when the staging is designed with more number of columns i.e. 10 for 600 KL capacity and 12 for 1000 KL capacity.

The solution of the staging for problem 2 are shown in the Table 7. Since 60 cm column with 3 percent of steel

was found unsafe with M200 mix , design with M250 is attempted. It violates the constraint GJ(4) for the stress limits, that is,

$$\frac{\sigma_d}{\sigma_{ad}} + \frac{\sigma_b}{\sigma_{ab}} \leq 1 \quad (\text{Refer PAGE 47}).$$

Instead of trying with larger diameter with the M200 mix, the design using M250 concrete mix is attempted. In the case of the staging for larger capacity containers it is better to go in for the richer mix for staging because it may reduce the weight of the staging which inturn will reduce the cost of the foundation. It is obvious from the solutions of 600 KL capacity problem, that the diameter of the column will become higher with the steel towards the minimum which increases the weight of the staging though its cost is reduced. It can be concluded that the staging design should be attempted for different mixes in order to pick up the better minimum design of the entire structure, keeping in mind that the foundation cost depends on the weight of the staging.

The solution of the foundation design problems are shown in the Table 8 . It can be seen that the minimization of the cost is achieved by the reduction of the slab width,  $X_5$ , reduction of the thickness of slab,  $X_4$  towards the minimum thickness reduction in the depth of the ring beam,  $X_2$ , and the reduction in steel variables. In problems F(600,6,4) and

Refer. Table 8

F(600, 8, 4) the thickness of the slab at the junction of the beam,  $X_3$ , is increased from the starting point value. This is because of the increase in soil pressure due to the reduction in the bearing area as the slab width decreases.

The variables  $X_6$  and  $X_7$  are the steel variables expressed as fraction of the cross section. It can be seen that in the solutions at the optimum point, the values are in the ratio of 2:1 approximately, though they are treated as two separate variables in the design vector. This behaviour is one which is expected because the negative and positive bending moments are also in the same ratio. In the formulation of the container problem one of these two steel variables (in ring beam 3) only is included in the design vector unlike both in the foundation problem. Thus the behaviour of the variables in the solution of the foundation ring beam assures the correctness of the formulation of the container with one steel variable in the ring beam 3.

It has been observed from the optimised values of the container that the total load transferred to the staging, and the distance at which equivalent earthquake force or wind force on the container, <sup>act</sup> are not ~~differing~~ <sup>different</sup> significantly for different number of columns or braces. ~~vary~~. From this it can be concluded that there is no interaction between the container

~~the~~ However there will certainly be interaction and <sup>^</sup>staging. ~~but it is there~~ <sup>^</sup>between staging and foundation. But in the present work the staging and the foundation are separately optimised and to that extent the results obtained are approximate.

#### Cost Vectors and Cost Matrices

##### 1. Container cost vector of 600 KL Problem:

Columns	Cost in Rs.
6	70721
8	62298
10	62329

##### 2. Container cost vector of 1000 KL problem:

Columns	Cost in Rs.
8	123,987
10	1,03,705
12	1,17,822

The cost values in the following matrices are in rupees.



## 3. Staging cost matrix of 600 KL Problem:

<u>Panels</u>			
Columns	3	4	5
6	16,054	18,535	20,150
8	15,990	17,630	19,930
10	15,820	18,875	23,247

## 4. Foundation cost matrix for 600 KL problem:

<u>Panels</u>			
Columns	3	4	5
6	18,297	16,987	17,090
8	10,459	9,420	6,232
10	12,320	11,340	10,200

## 5. Staging cost matrix for 1000 KL problem

<u>Panels</u>			
Columns	4	5	6
8	30,820	34,450	39,276
10	35,940	37,120	38,443
12	44,976	46,320	49,300

## 6. Foundation cost matrix for 1000 KL problem:

Panels	4	5	6
Columns			
8	38,229	37,625	22,070
10	37,245	36,725	24,092
12	21,941	20,895	20,624

## Total cost Matrix for 600 KL problem:

Panels	3	4	5
Columns			
6	1,05,072	1,06,243	1,09,961
8	88,747	89,348	88,460 (minimum)
10	90,469	92,544	95,776

## Total cost matrix for 1000 KL problem:

Panels	4	5	6
Columns			
8	1,93,036	1,96,062	1,85,333
10	1,76,890	1,77,550	1,66,240 (minimum)
12	1,84,729	1,85,037	1,87,746

From the total cost matrices it can be observed that the minimum cost design for 600 KL problem is the one in which the staging contains 8 columns and 5 panels. Similarly, the minimum cost design for 1000 KL capacity problem is the one in which the staging contains 10 columns and six panels. That is, the optimum design of the entire structure for 600 KL capacity problem can be had from  $C(600,8)$ ,  $S(600,8,5)$  and  $F(600,8,5)$ , and for 1000 KL capacity problem the optimum design can be had from  $C(1000,10)$ ,  $S(1000,10,6)$  and  $F(1000,10,6)$ .

### 5.5 Conclusions:

From the above discussions, the following conclusions are summarised.

1. The geometric variables in the container are not varying significantly to obtain the minimum cost.
2. The reinforcements in the container ring beams and the walls, and the reinforcement in the columns go to the minimum at the optimum design point.
3. Richer mix in the staging for large capacity container gives the better design.
4. For 600 KL capacity problem the design with 8 columns and 5 panels and for 1000 KL capacity problem the design with 10 columns and 6 panels are found to give the minimum cost designs.
5. The interaction between the container and the staging is observed to be less significant.

## 5.6 Proposed Extension of the Work:

The solution of any investigation ends with certain limitations and approximations. But the solution with acceptable mismatch from the exact solution can measure the efficiency of the investigation. In this work, the approximations and the limitations of the methods and the formulations are pointed out then and there. Following are the problems which can be investigated to get better study over this work, in a better computing environment.

1. The separate programs which are used to get the optimal design of entire structure can be coupled together to get the solutions in automate manner with the provision of selecting the proper type of foundation and use the same in the corresponding routine.
2. The space frame program can be developed and the approximate analysis of the supporting frame with the exact solution can be compared. The validity of the approximate analysis of the frame, with the variation of the number of columns and number of bracings can be investigated.
3. In the optimum design of the staging the panel heights are kept equal. The optimum cost of the staging with unequal panel heights can be investigated keeping the panel heights as design variables.

4. The number of columns and the number of bracings can be considered as integer variables and the solution can be sought using mixed integer programming algorithms.

5. Rather than carrying out substructure optimization and solving each one of them as NLP, the entire problem can be handled as a 3 stage Dynamic programming problem.

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## APPENDIX

Following are the cost values used in this present work:

Cost of concrete in the container (M200)	: Rs. 300 per m <sup>3</sup>
Cost of concrete in the staging (M200)	: Rs. 280 per m <sup>3</sup>
Cost of concrete in the staging (M250)	: Rs. 300 per m <sup>3</sup>
Cost of concrete in the foundation(M200)	: Rs. 260 per m <sup>3</sup>
Cost of steel	: Rs. 2800 per tonne
Cost of straight shuttering	: Rs. 20 per m <sup>2</sup>
Cost of conical shell shuttering	: Rs. 25 per m <sup>2</sup>
Cost of spherical shell shuttering	: Rs. 30 per m <sup>2</sup>